Elements of an Aesthetic Universe

Systematisation of Atonality and Dissonance
in Amotivic Serial Music Composition

Dissertation submitted in partial fulfilment
of the requirements for the degree of
Doctor in the Arts

by

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2015

"Ich fühle Luft von anderem Planeten."

Stefan George[1]

"Es klingt alles wie aus einer anderen Welt herüber."

Gustav Mahler[2]

"Une image poétique peut être le germe d'un monde,
le germe d'un univers imaginé devant la rêverie d'un poète."

Gaston Bachelard[3]

[1] Stefan George, Entrückung 1907. in: *Gesamt-Ausgabe der Werke. Endgültige Fassung*. Georg Bondi, 1931, Vol. 6/7: Der siebente Ring, p. 122.
[2] Gustav Mahler, in Herta Blaukopf (ed.), *Gustav Mahler Briefe: 1879-1911*. Rev. and enl. Ed. Publications of the International Gustav Mahler Society, Vienna/Hamburg, 1983, p. 142, letter to Arnolf Berliner of 31 January 1895 quoted in: Constantin Floros, *Gustav Mahler, The Symphonies*, Breitkopf & Härtel, 1985, translated by Vernon & Jutta Wicker, Amadeus Press, 1993, p. 51.
[3] Gaston Bachelard, *La poétique de la rêverie,* Presses Universitaires de France, 1960, p. 1.

Abstract

The present doctoral dissertation provides the description of artistic research on Chromatic Interval Group serialism (CIG-serialism), a technique developed by the author in 1997. The aim of this structurally amotivic technique is to compose music that is highly atonal and dissonant in a systematic way. It starts from Reginald Smith Brindle's idea of "atonal series", which allegedly maintain a constant high degree of atonality. These series consist entirely of what will be called 'chromatic interval croups of order 3', or CIG-3's (ordered pitch class sets containing three pitch classes, at least two of which are interval class 1 apart). The aim of the present research is to assess Smith Brindle's claim and to find out whether it is possible to adapt the CIG technique in order to enhance the desired result of systematic atonality and dissonance.

In Part 1 of the text, after a descrition of the original CIG technique, the concepts of tonality/atonality and consonance/dissonance are discussed and (re-)defined in a manner that makes quantification of both musical aspects possible. Two formulas—one for the quantification of tonality and one for the quantification of prime consonance—are developed. With these formulas, the degree of tonality and prime consonance of pitch class sets belonging to any set class can be determined. Methods of tonality and consonance analysis of music based on the twelve pitch classes of the chromatic scale are subsequently developed. Part 1 culminates in the assessment of CIG-3-serialism and its further development into general CIG-serialism. In a general CIG-series, all groups of consecutive pitch classes of any size form CIG's of any order (not just order 3). It is demonstrated that general CIG-series systematically yield music with the highest constant degrees of atonality and dissonance.

Part 2 of the dissertation covers issues of an aesthetic nature. It introduces the concept of 'aesthetic universe' of an artist, and defines artistic practice as the expression of ideas belonging to this aesthetic universe. The aim of Part 2 is to show that CIG-serialism is the technique that is indispensible to express the ideas of what will be called the 'idiosyncratic part' of the author's personal aesthetic universe. The formulas developed in Part 1 describe the 'endophysical laws' of this aesthetic universe.

Part 3 provides a description and analysis of the artistic output of the present artistic research, the seven compositions that together form the 'Elements Project'. It is the result of the synthesis of Parts 1 and 2. Part 3 shows how the compositions of the Elements Project are the expression of the metaphorical Empedoclian Elements (earth, water, air and fire) of the author's aesthetic universe. The artistic output consists of the following three central orchestral pieces and four 'complementary' pieces:

Danse de la terre	for orchestra
Danse de l'eau et de l'air	for orchestra
Danse du feu	for large orchestra
Le sourire infini des ondes	for ensemble (9 instruments)
Un souffle de l'air que respirait le passé	for piano quartet
A l'image du monde… originel	for piano
A l'image du monde… double	for piano

Preface

When I started my doctoral research at the Orpheus Institute and the University of Leuven in 2009, I thought it would be about ‘the systematisation of atonality and dissonance in amotivic serial music composition’ and about a serial technique I had been working with for over a decade. Although this proved to be the case it turned out that musical theory and compositional technique were only part of the game. As time went by it became more and more clear that my research was also about myself, about the ‘world’ I live in as a composer; not the physical world that surrounds me, but the cerebral world of what I think, what I feel, and who I am. My research turned out to be about what I termed the ‘aesthetic universe’; about my own aesthetic universe and about aesthetic universes in general; about what makes artists the individuals they are; about what artistic practice and artistic research are. The initially planned title of my dissertation was therefore degraded to become a subtitle and the project became even more ambitious than it was at the start.

Although the most extended part of the present dissertation covers the concepts tonality and consonance and their quantification, these concepts have to be understood within the broader theory of the aesthetic universe of which the formulas that are developed in the text represent endophysical laws. Of course, the research described in the present text is not only about me, because that would reduce its relevance considerably. It is not exclusively about my personal technique either. Indeed, although the concepts of dissonance and (especially) tonality used in the present dissertation, as well as the technique that is assessed and refined in my research, are highly idiosyncratic, they are placed in a broader, more general musical and aesthetic context. This becomes apparent in the methods of analysis (T-analysis and PC-analysis) developed in Part 1 and in the aesthetic ideas discussed in Part 2. Part 3 is the synthesis of the ideas developed in the two preceding parts.

I gladly and proudly express my gratitude to everybody who has contributed to my research and to the realisation of this dissertation in one way or another; in the first place to prof. Dr. Mark Delaere, the supervisor of my doctoral research, for his expert advise on academic as well as artistic topics; to my co-supervisor and colleague composer Luc Van Hove; to my advisors prof. Dr. Raf Cluckers, the late Dr. Bob Gilmore, and Luca Francesconi, but also to Marc Erkens, the head of the music department of the LUCA School of Arts for his unremitting confidence in me and for giving me the opportunity to accomplish a master trajectory at my not-so-youthful age in the run-up to my doctorate; to André Laporte, my composition teacher at the Royal Music Conservatory of Brussels, for being one of my artistic fathers; to all my artistic research partners-in-crime at the ORCiM and in the docARTES-programme (Orpheus Institute, Gent), to Peter Dejans, director of the Orpheus Institute, to Dr. Kathleen Coessens, Dr. Marcel Cobussen, Dr. Vincent Meelberg, and especially to Dr. Luk Vaes, for being an example to me of how artistic research can be at the same time artistically enthralling and scientifically rigorous; Thanks to the LUCA (formerly: FAK, Dr. Peter De Graeve) for the scholarship that enabled me to dedicate all my time to my research during three years, and to prof. Dr. Gerhard Nierhaus of the KunstUni Graz. Thanks also to the performers of the artistic output of my research: Filip Rathé and the Spectra Ensemble, Tetra Lyre, Stefan Blunier and the Belgian national Orchestra, Simeon Pironkoff and the Anton Webern Symphony Orchestra, pianists Eva Bajic and Jan Michiels, and the Goeyvaerts String Trio. Special thanks to Willy Dillen for meticulously reading the aesthetic part of the text; to the many friends and colleagues whose names I forget to mention here; and last but not least to my wife Johanna and my two daughters, Lotte and Stella, for their eternal patience and tolerance during the years that I was physically present in our house, but mentally absent from our home when my energy and full attention were claimed by the research of which the present text is the written testimony.

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List of abbreviations and usages

CIG	chromatic interval (class) group
ic	interval class
mod	modulo
pc	pitch class
rev.	revision or revised
RHS	rhythmic-harmonic substructure
s.a.	without year indication (sine anno)
T	tonality (in T-formula, T-analysis, T-graph,…)
PC	prime consonance (in PC-formula, PC-analysis, PC-graph,…)

Forte numbers of set classes (and pc-sets belonging to those classes) are written between square brackets (e.g. [6-14]). Whenever a pc-set is called by its (extended) Forte name, it should be understood that what is meant is that the pc-set is *an instance of* the set class with that name.

Pitch class content of pc-sets is written with pitch class numbers between square brackets separated by commas (e.g. [0,1,2]).

Interval class sets are written as a list of digits separated by commas between round brackets (e.g. (1,3)).

Interval vectors are written as a string of 6 un-separated digits between hooked brackets (e.g. <013458>).

Deviations from traditional pc-set theory nomenclature:

> - The concept 'set class' is used in a slightly different way than is done in Alan Forte's Pitch Class Set Theory: whereas Forte's set classes are determined by all pitch class sets *and their inversions* with the same prime form, in the present dissertation inversions and un-inverted pitch class sets form distinct set classes (actually sub-classes of forte's set classes) whenever they have different prime forms. Set class [3-2], for instance, is distinguished from its inversion [3-2i]; together they form Forte's set class [3-2].
>
> - Numbers for set classes without Forte-numbers: interval classes ([2-1] through [2-6]), and set classes of cardinality 10 ([10-1] through [10-6]), 11 ([11-1]) and 12 ([12-1]).

I opted to always quote in English, as is customary in English academic texts, even when the quoted source is originally written in another language (with the exception of the epigraphs). Whenever translated quotes occur in the text, the original version is added (in italics) in a footnote, with the exception of canonical quotes such as the one by Goethe in chapter 2.

The text features several neologisms (such as 'amotivic' and 'endophysical') that were necessary for want of existing vocabulary. All neologisms that are used as technical concepts are defined or explained at first occurrence in the text.

Italicisation appearing in quotations is always that of the original author, except when the indication '[my italics]' is added.

Introduction

1. Research questions

The present dissertation is the result of artistic research on the compositional technique I developed in 1997, which I called Chromatic Interval Group Serialism (CIG-serialsm). The aim of this technique was to compose amotivic music that is highly atonal and dissonant in a systematic way. At least that was the assumption. The first goal of my research was to confirm the claim that CIG-serialism does indeed yield highly atonal and dissonant music in a systematic way. If this confirmation proved to be possible, the second goal was to find out whether the technique could be improved in order to obtain music that was even more atonal and dissonant.

These envisaged goals generated the research questions of my artistic research. The first central research question results from the first goal: 'Does the technique of CIG-serialism yield highly atonal and dissonant music?' This central question leads to several secondary research questions: 'What is atonality?' 'What is dissonance?' 'If certain pieces can be called more (ore less) atonal or dissonant than others, is it possible to quantify those aspects? How can dissonance and atonality be quantified?' The second goal provokes the second central research question: 'How can the technique of CIG-serialism be altered, adapted, or refined in order to achieve an idiom with an even higher degree of atonality and dissonance throughout (i.e. resulting in music that is even more atonal and dissonant) whilst preserving the idiosyncrasies of my personal style?'

2. Aesthetic incentives

My search for music that is highly atonal and dissonant as well as amotivic is the result of my personal aesthetic convictions, which aim at the highest possible degree of aesthetic 'pureness'. I try to reach this goal through a directed choice of idiomatic and stylistic elements aiming at the strongest possible coherence of musical materials; this can be done, for instance, by avoiding the blend of tonal and atonal elements or by aiming at music that has a constant degree of consonance.

In 1997, I realized that the existing techniques were inadequate for the expression of my idiosyncratic aesthetic ideas. A new technique was required; one that reflected the endophysical laws of my aesthetic universe more accurately, whilst preserving the idiosyncrasies of my personal style.[4] CIG-serialism seemed to be the outcome of the search for such a technique that was the most suitable and workable to achieve my objectives.

Like dodecaphony or tonal harmony, CIG-serialism too is not more than a technique. A technique is a tool; it is a procedure necessary to express the ideas of an aesthetic universe. What I want to express in my music is not (or at least not merely) a technique. Technical aspects may (and do) belong to the aesthetic message I convey, just like grammatical aspects may (and do) belong to the message conveyed in a poem; but to reduce a message to the technique used to express it would be an undervaluation of the aesthetic value of a work of art.

I am well aware of the fact that the CIG-serialism as a means to obtain technical and idiomatic pureness is necessarily limiting my choices as a composer, but in this respect, CIG-serialism does not differ from other compositional techniques or idioms, including the tonal ones. Limitation is indeed central to any form of composing. A composer can only create a musical structure that can be called a 'composition' by choosing certain elements, and thereby excluding all others. Without making that choice, without imposing restrictions, there is no composition, no matter how strict or flexible those

[4] The concepts of aesthetic universe, aesthetic idea, endophysical law, and related concepts will be amply discussed in Part 2.

restrictions may be. John Cage's piece *4'33"* (from 1952), for instance, is based on only two well-specified constraints: the duration of the piece and the actions performed (or rather not performed) by the performer(s). The piece is limited in time to a duration of exactly four minutes and thirty-three seconds—hence the title. In addition, the score just prescribes that the performers do not play their instrument throughout the entire three movements of the piece. The performers are therefore, just as is the case in any other composition, not entirely free in their actions. Without the presence of these restrictions *4'33"* could not be called a fully-fledged composition.

Composers of any stylistic or idiomatic predilection—whether tonal or serial or whichever stylistic or idiomatic group—pick the elements they want to use and the elements they want to avoid in their compositions. The purer they want to keep their idiom, the more stringent and targeted the restrictions they impose upon themselves will be. Despite persistent general opinions, the choices made in what is traditionally called tonal music, are usually no less restrictive than those of serial composition. Tonal music is mostly limited to the local use (during a shorter or longer duration within a piece) of 'only' seven of the twelve pitch classes of the chromatic scale (the so-called diatonic sets)[5], whilst most serial techniques exclude no pitch class (sometimes they don't even limit themselves to those of the chromatic scale). In terms of the simultaneous use of the pitch classes (in chord formation) the limitations of tonal music are even stricter. Traditional tonal chords are formed exclusively by stacking minor and major thirds on top of one another. This limits the tonal composers to only four of the nineteen possible triads (the tonal triads) and eight of the forty-three possible four-note chords (the seventh chords). Chords containing more than four pitch classes are very rare in tonal music—at least until the end of the nineteenth century—since they reduce the perception of tonality and are situated at or beyond the border between tonality and atonality. And if, in rare cases, ninth chords occurred in tonal music, their use was vigorously restricted for centuries, as is rather painfully illustrated—painful to most 21th century musicians at least—by the following example.

When Arnold Schoenberg composed his string sextet *Verklärte Nacht* op. 4 back in 1899, inversions of ninth chords were strictly forbidden and 'therefore' nonexistent, as Schoenberg describes in his seminal *Harmonielehre*.[6] The Viennese *Musikverein* refused to programme the sextet because of a single inversion of a ninth chord it contains (in the example below, the inverted ninth chord is indicated with an asterisk).

bars 41 and 42 of Arnold Schoenberg's *Verklärte Nacht* Op. 4
(source: Arnold Schoenberg, *Theory of Harmony*, p. 346).

Schoenberg remarked, not without a healthy dose of cynicism:

> Only now I understand the objection, at that time beyond my comprehension, of that concert society which refused to perform my Sextet on account of this chord (its refusal was actually so explained). Naturally: inversions of ninth chords just don't exist; hence no

[5] The term 'diatonic set' is here used in its larger sense including pc-sets of set classes [7-32], [7-34] and [7-35].
[6] Arnold Schoenberg, *Theory of Harmony*. translated by Roy E. Carter, Faber & Faber, 1983.

> performance, either, for how can one perform something that does not exist.[7]

In sixteenth-century music, even tighter restrictions were prevalent than in later tonal music. Josquin Desprez and his contemporaries squeezed into a very tight straitjacket of musical restrictions. However, this does not diminish the artistic value of their music. On the contrary, Goethe's *"In der Beschränkung zeigt sich erst der Meister"*[8] may be regarded as a quality label. This applies to serial music as much as to tonal music. To create valuable art with limited resources is an artistic endeavor of all time.

The difference between my approach—which may be considered to be part of the ideas of (neo-) modernism[9]—and its more traditional counterpart lies in the fact that, in my idiom based on a serial technique, I am consciously exploring aesthetic boundaries in order to expand my aesthetic universe, and the possible aesthetic universe of the culture that is willing to adopt my music and the aesthetic ideas it expresses. I strive for independence in that area but realize that I can never ignore tradition. My choices are necessarily coloured by influences from the culture that I carry with me.[10] That does not mean that it is impossible to move aesthetic boundaries in an imaginative, personal and meaningful way. I just feel no obligation to adopt the artistic restrictions determined by the compositional practice of the past unchanged and unchallenged. Such obedience to tradition would be, according to Gustav Mahler, mere "sloppiness"[11].

In addition to my search for purity, the search for music that forms an organic sounding whole is a second major aesthetic endeavour in my practice as a composer. Striving for organic coherence in serial composition assumes that the resulting musical works transcend their strictly serial substructure. In this respect, I like to compare my approach with the principles of genetics: just as living organisms are highly (but not solely) determined by their genetic material, my compositions are highly (but not solely) determined by their series. The series not only provides for the pitch material but also directs the course of the entire structuring and transforming process leading to a piece of music, comparable to the biochemical processes that transform genetic material (the organism's genotype) into the ultimate living organism (its phenotype). But just like living beings transcend their genetic material, my compositions (the musical phenotype) are more than the series (the structural genotype) on which they are based. In this creative process, which to a large extent is based on intuitive aesthetic sensitivity or taste, the serial technique is not more decisive than the way it is implemented. Technique and artistic taste cannot be considered separately from each other but should complement each other in a constant interaction. Strictly adhering to rules does not guarantee artistically valuable results; aesthetic transcendence is un-dispensable. In this respect again, serial techniques are in no way different from the techniques used in tonal composition.

The main goal of my quest for an answer to the research questions stated above is to find a way to increase the aesthetic pureness and organic coherence of my music. An adaptation of CIG-serialism that yields music with the highest possible levels of dissonance and atonality in a systematic way may further enhance the rigorousness of my idiom and restrict the gamut of possible choices even more, but

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7 Arnold Schoenberg, *Theory of Harmony*, p. 346.

8 Johann Wolfgang von Goethe, *Sämtliche Werke, Jubiläums-Ausgabe in 40 Bänden*, Vol. 9. re-edited by Eduard von der Hellen, Cotta o. J., 1906, p. 235. The quote literally translates as "It is in restrictions that the master reveals himself", which is more commonly phrased as "less is more".

9 Artistic modernism is an aesthetic movement, tendency or conviction that searches for aesthetic innovation, in order to push (cultural) aesthetic boundaries. Historically, an idea of progress, improvement and truth has been associated with modernism since the Enlightenment; an idea that, in my opinion, no longer seems tenable after postmodernism. That's why I prefer 'neo-modernism' as an epithet for my idiom and style, since it distinguished between the idea of innovation and the idea of progress.

10 Johann Sebastian Bach, Johannes Brahms, Gustav Mahler, Luigi Nono, and Pierre Boulez are amongst the composers that probably most influenced my aesthetic thought.

11 *"Tradition ist* Schlamperei" Quoted in Kirk Ditzler, Tradition ist "Schlamperei". Gustav Mahler and the Vienna Court Opera. in *International Review of the Aesthetics and Sociology of Music*, Vol. 29, N°1, June 1998, p. 11. Charles Rosen phrases it even more strongly: "The name generally given to widely accepted error is *tradition"* (Charles Rosen, *The Frontiers of Meaning, Three Informal Lectures on Music.* Kahn & Averill, 1994, p.11) [Rosen's italics].

I am convinced that these restrictions are only of a technical nature, and that my aesthetic possibilities and artistic freedom will only be stimulated and intensified.

My serial technique may be cerebral—and even more so when it is made more rigorous—but that does not mean the music that results from it is not more than a product of the brain, lacking all expressive power and emotion. Serial music is not necessarily less expressive, or no less 'coming from the heart' than more 'intuitive' tonal music. Each composition is a product of cerebral effort. The thought processes of composing, irrespective of the style or idiom or the technique used, are partly conscious but also escape to some extent conscious control. It is these uncontrolled processes that are said to come 'from the heart'. Both conscious and unconscious cerebral processes provide organic structure, coherence, and consistency of a composition.

Besides constrains, structure is a second indispensable cornerstone of composing. Without structure, there can be no question of a composition. Igor Stravinsky noted in this context: "Music's exclusive function is to structure the flow of time and keep order in it."[12] Strictly designed structure is no impediment to expressive power however. Musical expression is a controversial concept. Stravinsky claims that music is not able to express anything at all. He writes:

> I consider that music is, by its very nature, essentially powerless to express anything at all, whether a feeling, an attitude of mind, a psychological mood, a phenomenon of nature, etc....Expression has never been an inherent property of music. That is by no means the purpose of its existence. If, as is nearly always the case, music appears to express something, this is only an illusion and not a reality. It is simply an additional attribute which, by tacit and inveterate agreement, we have lent it, thrust upon it, as a label, a convention - in short, an aspect unconsciously or by force of habit, we have come to confuse with its essential being.[13]

On the other hand, if we restrict the term 'expression' to 'emotional expression'—the kind of expression Stravinsky was probably referring to[14]—one could argue that there is no music that is *not* expressive; that all music has the potential to express something. Just like every object of communication, whether it is a poem, a statement or a facial expression, music is potentially expressive. That expression is subjective, relative and culture-bound. Musical expression is subjective because each listener reacts in a different way on the musical stimuli, and the response to these stimuli depends on the context in which the music is heard. The expression is relative and culture-specific because it depends on the relationship between the listener and the music. This relationship is personal and is partly due to the familiarity of the listener with the culture the music belongs to.

3. Methodological observations and structure of the dissertation

The following text is structured in three parts. Part 1 (Atonality and Dissonance) treats theoretical aspects. Part 2 (the Aesthetic Universe) covers aesthetics. Part 3 (the Elements Project) describes the artistic output of my research.

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[12] Quoted in Géza Szamosi, *The Twin Dimensions: Inventing Time and Space*, McGraw-Hill, New York, 1986, p. 232. This puts Stravinsky in line with Eduard Hanslick who wrote: "The content of music is tonally moving forms." (Eduard Hanslick, *On the Musically Beautiful; A Contribution towards the Revision of the Aesthetics of Music*, trans. Geoffrey Payzant, Hackett Publishing Company, 1986, p. 29).

[13] Igor Stravinsky, *An Autobiography*, W.W. Norton & Company, 1962, pp. 53-54.

[14] In the broader sense of the term 'expression' used in the present text, *all* art is expression (of ideas belonging to the artist's aesthetic universe). See Part 2.

Chapter 1 is a description of the technique of CIG-serialism, based on the exclusive occurrence of CIG's of cardinality 3 in the construction of the series, as it was developed in 1997. It is the starting point of the research. After the research questions have been restated in Chapter 2 in the light of the ideas delineated in the previous chapter, the concepts of tonality (Chapter 3) and consonance (Chapter 4) are discussed. First, traditional definitions of both concepts are stated. This is not done in a comprehensive manner. The topic is only elaborated for as far as it is relevant in the present context of artistic research, and not as a musicological subject. Therefore, the descriptions are at times sketchy. The concepts of tonality and consonance are defined in the light of their contrasts atonality and dissonance, and are re-defined in such a way that quantification becomes possible. For that purpose, formulas to determine the degrees of tonality and (prime) consonance of pitch class sets belonging to any set class are developed. On the basis of these formulas, analysis methods are developed that can be implemented in any music based on the twelve pitch classes of the chromatic scale in equal temperament.

Chapter 5 assesses CIG-serialism, using the formulas constructed in Chapters 3 and 4 as a starting point. It will be shown that the initial assumption (that CIG-serialism yields highly atonal and dissonant music) stands, but that the result can be enhanced if the technique is further restricted. This results in the adapted technique that will be called 'general-CIG-serialism', which is based on the use of series that are exclusively constructed with CIG's of any cardinality (and not just cardinality 3, as is the case in the original CIG-technique).

The two chapters of Part 2 (Chapters 6 and 7) cover the aesthetic topics of knowledge and (artistic) communication, and introduce the concept of aesthetic universe. In this second part, I show how the formulas developed in Part 1 describe the endophysical laws of my personal aesthetic universe, and how my technique is the code necessary to express the ideas belonging to the idiosyncratic part of that cerebral universe. Part 2 is meant to bridge the gap between the theoretical first part and the 'artistic' third part, which provides a description of the artistic output of my research. In this third part, I show how my music is the expression of the *what* and the *how* of my aesthetic universe.

Part 1 will mainly (but not exclusively) be situated on a syntactic level. I will show that the technique assessed and developed consists of syntactical rules internal to the procedural knowledge of artistic expression. Part 2 belongs predominantly to the semantic sphere. It treats meaning of conceptual knowledge and its link with the expressive procedures of aesthetics. Part 3 focuses on the artistic synthesis of syntax and semantics; the blending of concepts and mental images of aesthetic procedures in the act of artistic creation.

Part 1
Atonality and dissonance
Assessment of an amotivic serial composition technique

Chapter 1. 54-CIG serialism

1.1 Genesis of chromatic interval group serialism

1.1.1 Prehistory

In my compositions of the first half of the nineties I resolutely opted for a dissonant and atonal sound idiom. This resulted in compositions using predominantly interval class 1 (ic 1) around one or several tone centres, presuming that ic 1 is the most dissonant interval.

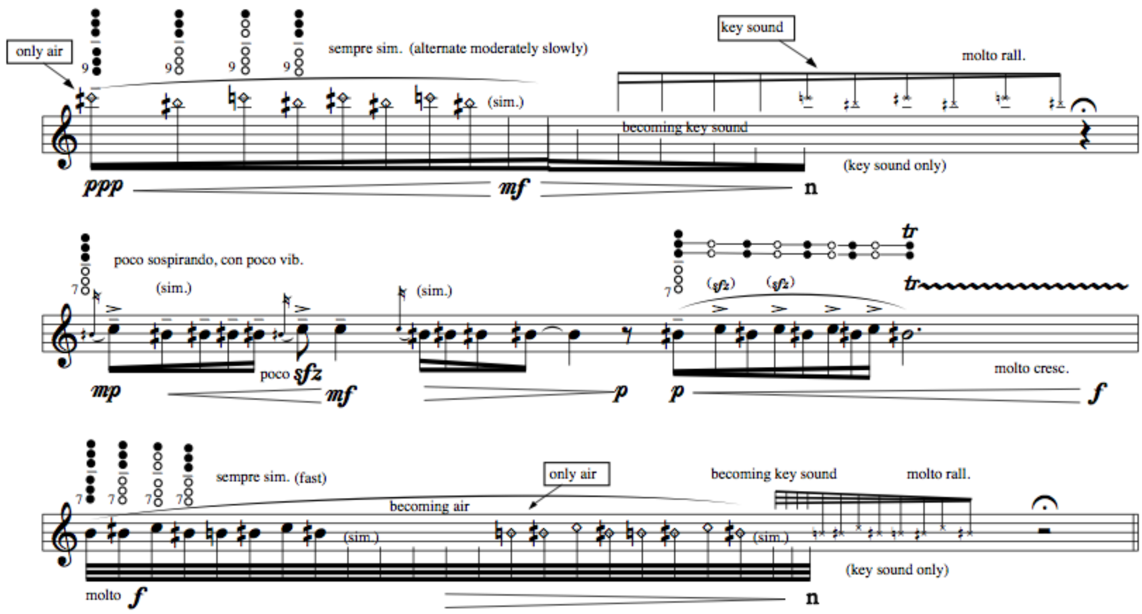
Monodie for piano solo (1992; second version 1995) is a clear example of this (see Example 1.1). The whole piece is based on the central pitch class A in different registers of the piano. Around this central note or pitch class I constructed clusters of varying 'thickness'[15], the frequency of occurrence of which is based on a Gauss distribution (the thickest clusters being the least frequent). The clusters have an ornamental function[16], not a structural one; they ornament or embellish the central pitch class.

[15] The thickness of a cluster is the number of notes the cluster is made of. The term 'cluster around a pc' is here not used in the strict sense where not a single pitch class may be left out inside the cluster interval. In some clusters of *Monodie,* even the central pitch class (A) is absent, as can clearly be seen in the excerpt from Example 1.1.

[16] "The musical arabesque, or rather the idea of 'ornament', is the basis of all forms of art" ("l'arabesque musicale' ou plutôt le principe de 'l'ornement' est la base de tous les formes d'art"). Claude Debussy, *Monsieur Croche et Autres Récits*, 2nd edition, Gallimard, Paris, 1971, pp. 33-34 [my translation].

Example 1.1: Excerpt from *Monodie* for piano solo.

A similar procedure was used in *La couleur du vent* for flute solo (1996) (see Example 1.2). The central pitch classes in this piece are C and F sharp. My concern at that time was how to justify the use of more than one tone centre in order to be able to use more than just ic 1. Which criteria can be applied to change from one central tone to the next if those central tones are not a semitone apart? Why go from C to F sharp (as in *La couleur du vent*) and not to F natural or G? How could I introduce intervals (or interval classes) that are less dissonant than ic 1 or have stronger tonal connotations (like the cadential feeling of perfect fourths and fifths), if I want to obtain highly dissonant and atonal music? How could I obtain more melodic variety and still keep my music atonal and dissonant?



Example 1.2: Excerpt from *La couleur du vent* for flute solo.

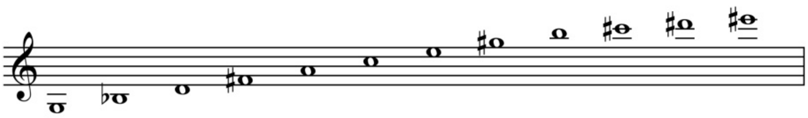
A first answer to the questions stated above was explored through the use of dodecaphonic series, for instance in *Tout près de l'eau* for mezzo-soprano and alto flute (1995). However, I was unsatisfied with this approach because of the discrepancy between the obtained idioms. The 'central-tone' approach yielded a sonic result that was too different from that of the traditional use of dodecaphonic techniques. The former had a more static, stable melodic and harmonic shape than the latter. This melodic and harmonic instability was not in accordance with my aesthetic preferences and goals. A second objection to the technique of dodecaphony was the fact that dodecaphony does not necessarily result in atonal music. The explanation for this claim brings us back to the origins of dodecaphony.

When Arnold Schoenberg developed the dodecaphonic technique in 1921-1922, he was looking for a way to organise music independently from the principles of tonality. René Leibowitz wrote in this respect:

> The twelve-tone technique is an essential discipline within atonality, an organization containing structural elements powerful enough to replace the tonal system.[17]

[17] "La technique de douze sons constitue au sein de 'l'atonalité' une discipline indispensable, une organisation contenant des éléments structurels suffisamment puissants pour remplacer ceux du système tonal". René Leibowitz, *Introduction à la musique de douze sons: Les variations pour orchestre op. 31, d'Arnold Schoenberg*, Editions L'Arche, Paris, 1949, p. 27.

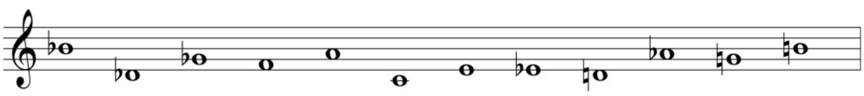
Schoenberg's pupil Alban Berg, however, showed in practice that, although dodecaphony may be an alternative for tonal thinking, it does not necessarily lead to atonal music. Boulez writes that "avoiding tonality"[18] (or as Leibowitz says: "the suspension of tonal functions"[19]) was Schoenberg's first aim, but if this is true, Schoenberg seems to have failed since it remains possible to compose music that is closely related to tonal music with the dodecaphonic technique, by using sets containing predominantly tonal elements—such tonal triads, or diatonic scales—or as a result of the way in which the series are treated within the composition. Berg's *Violin Concerto* (1935) is a striking example: the series on which this concerto is based (see Example 1.3) contains several of the mentioned tonal elements.



Example 1.3: Series of Alban Berg's *Violin Concerto*.

Not only do the triads of g (minor triad, notes 1 to 3), D (major triad, notes 3 to 5), a (minor triad, notes 5 to 7), and E (major triad, notes 7 to 9) occur successively in this series, with a clearly perceived tonic-dominant relation between the triads of g and D, and between the triads of a and E, but on top of that, the first seven notes of the series constitute the complete set of notes of the ascending melodic minor scale of g. Similarly, notes 3 through 9 form the set of the ascending melodic minor scale of a. In addition, the last four notes belong to a whole-tone scale that allows for the integration of the Bach Chorale *Es ist genug* from the cantata *O Ewigkeit, du Donnerwort* BWV60 (1723) that starts with this whole tone pattern. These elements constitute decisive reasons why Berg's **Violin Concerto** can be said to be closely related to tonality, although it is basically a dodecaphonic composition.

Dmitri Shostakovitch's *String Quartet N°13* in B flat major op. 138 (from 1969), as an example of tonal use of dodecaphony within a totally different idiom, is based on the series shown below (Example 1.4). The series is presented by the viola at the beginning of the quartet.



Example 1.4: Series of *String Quartet N°13* by Dmitri Shostakovitch.

In the viola introduction, the series is played in groups of four notes. Because of the positioning of the G flat (note 3 in the series) just before F, and as a result of the way it is presented in the quartet, this G flat is perceived as an appoggiatura for F. Therefore, the first group of four notes is perceived as a minor triad (B flat, D flat, F) with appoggiatura or added sixth (G flat). In the same manner, the second group of four notes in the series can be perceived as a diminished triad (A, C, E flat) with appoggiatura E (or F flat, note 7 in the series) for E flat. When this second appoggiatura is left out of the picture, the remaining first seven notes (from note 1, B flat, to note 8, E flat) constitute the complete set of the key of B flat harmonic minor. Therefore, this series is closely related to the tonal system; a fact that is amply exploited by Shostakovitch. Not only does the series contain tonal elements—like the series of Alban Berg's *Violin Concerto*—but the free way in which Shostakovitch treats the material contained in the series in the elaboration of the *String Quartet*—with his explicit predilection for tonal triads,

[18] Pierre Boulez, *Relevés d'apprenti*, Editions du Seuil, 1966, p. 16. "…la volonté première de Schönberg qui est "d'éviter" la tonalité".
[19] René Leibowitz, *Introduction à la musique de douze sons*, p. 112.

consonance, and diatonicity—results in music that is much closer to the structural principals of classical tonality than to the system envisaged by Arnold Schoenberg.

1.1.2 The atonal series

From the examples stated above it is clear that a dodecaphonic series does not automatically result in atonal music provided that it contains enough 'tonal elements'. George Perle calls those elements "non-dodecaphonic elements"[20]. He claims that a set "may be specially constructed to incorporate features not normally associated with the twelve-tone system"[21]. Atonal music can only be obtained if those elements are avoided in the construction of the series. In his book *Serial Composition*[22], Reginald Smith Brindle describes the construction of what he calls 'atonal series'. An atonal series, he says, is "a series which maintains throughout the same degree of atonality"[23]. Atonality is, according to him, a feature of "music which is not clearly organized by traditional systems, such as the modal system, or the major and minor key systems"[24]. To construct an atonal series, Smith Brindle claims, one has to eliminate the following elements in the series:

- Tonal triads
- Whole-tone relationships
- Successions of fourths (which produce a cadential effect)
- Chromatic chords (such as diminished seventh chords)[25]

According to Smith Brindle, this can be done if a series is constructed exclusively with the notes of what I call **chromatic pitch class sets**[26] (chromatic pc-sets) of order 3. A chromatic pc set of order 3 is a pc-set containing three pitch classes, at least two of which are ic 1 apart.[27] There are only nine of these chromatic pc-sets in prime form (nine chromatic set classes)[28] , as shown in Example 1.5 below:

Example 1.5: Representation of the prime form of all chromatic set classes.

According to Smith Brindle, every three successive notes in an atonal series should form a chromatic pc-set. He gives the example of Anton Webern's *Symphonie* op. 21 (composed in 1928) to illustrate this idea. From his opus 17 on, the dodecaphonic series of Webern's works consisted mainly of chromatic pc-sets. The series of the *Symphonie* op. 21 is entirely constructed with these pc-sets, as is shown in Example 1.6. Every three successive pitch classes within the series (pc's 1 to 3, but also 2 to

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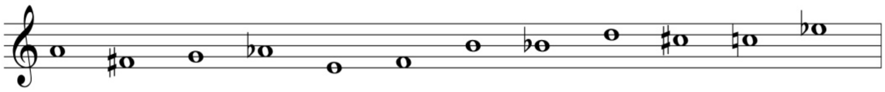
[20] George Perle, *Serial Composition and Atonality, An Introduction to the Music of Schoenberg, Berg, and Webern,* University of Califormia Press, 6th revised edition, 1991, p. 78.
[21] George Perle, *Serial Composition and Atonality,* p. 78.
[22] Reginald Smith Brindle, *Serial Composition*, Oxford University Press, 1966.
[23] Reginald Smith Brindle, *Serial Composition*, p. 12.
[24] Reginald Smith Brindle, *Serial Composition*, p. 12.
[25] Reginald Smith Brindle, *Serial Composition*, p. 12.
[26] Smith Brindle uses the term "note-groups of a chromatic nature" and "'atonal' note groupings" (Reginald Smith Brindle, *Serial Composition*, p. 12).
[27] The first value in the interval vector of a chromatic pc set of order 3 is at least "1". (The "'interval vector' of a pc set is an ordered 6-tuple of the multiplicities of intervals [interval classes] 1, 2, 3, 4, 5, 6 in that order" (John Rahn, *Basic Atonal Theory*, Schirmer Books, 1980, p. 100). It indicates how many times each of the six interval classes occurs in the pc set).
[28] The chromatic set classes have Forte numbers [3-1] through [3-5]. The prime form of a pc-set is its standard representation. It is the most compact form of the set with pc 0 (pitch class C) as its base. For more details: see a.o. Allen Forte, *The Structure of Atonal Music*, Yale University Press, 1973, pp. 3-5, and Joseph N. Strauss, *Introduction to Post-Tonal Theory*, Pearson Prentice Hall, 3rd edition, 2005, pp. 57-59. In the present text I extend the meaning of prime form to set classes and their inversions.

4, 3 to 5, etc.) constitute a chromatic pc-set. The series of the Webern's *Symphonie* op. 21 is therefore an atonal series according to Smith Brindle.



Example 1.6: Series of *Symphonie* op. 21 by Anton Webern.

1.1.3 Construction of the series

Since it was my goal to develop a technique that would systematically result in atonal music, starting from Reginald Smith Brindle's claim concerning the construction of atonal series, I looked for a way to construct series that, like those of Anton Webern's later works, only contain chromatic pc sets of order 3.

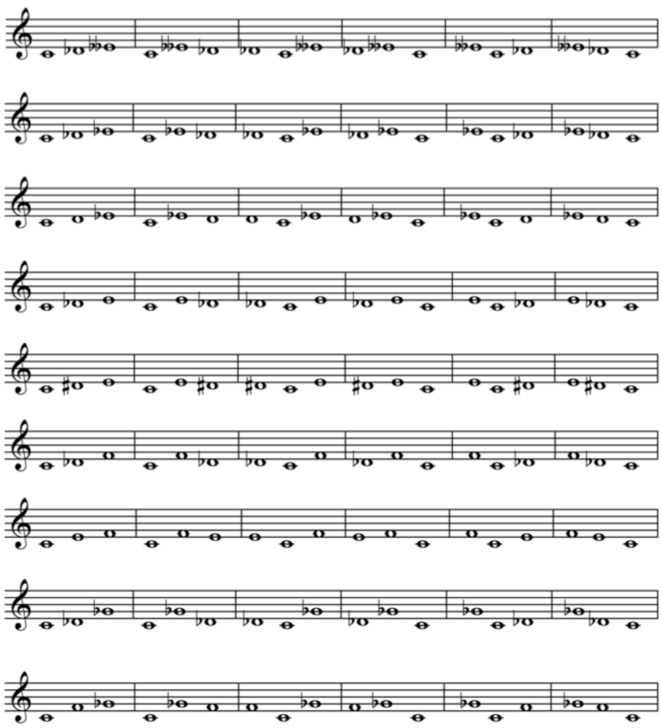
Each of the nine chromatic set classes can be turned into an ordered set in six ways (six permutations). I called those ordered sets derived from chromatic set classes of order 3 **chromatic interval groups**[29] (where 'group' stands for 'ordered set') or **CIG**'s.[30] Example 1.7 b shows the six CIG's that can be obtained by ordering the notes of the prime form of the chromatic pc-set belonging to set class [3-1] shown in Example 1.7 a. Any transposition of these six permutations is an instance of the same CIG. In other words, the pitch classes in a CIG are unimportant. What counts is the intervals between the three successive pitch classes. Therefore the ordered group is called a chromatic *interval* group, and not an interval *pitch* (class) group.

Example 1.7 a: Chromatic pc-set (belonging to set class) [3-1].

Example 1.7 b: Six permutations of a chromatic pc-set [3-1].

The fifty-four possible CIG's resulting from the six permutations of each of the nine prime forms of chromatic set classes are shown in prime form in Example 1.8 below.[31]

[29] More precisely, it should be called a chromatic interval class group.
[30] After the assessment of CIG-serialism in the present dissertation, it will be necessary to specify the order of the CIG, since there will appear to be CIG's of higher order as well. What is called a CIG here will turn out to be a CIG of order 3 (or CIG-3), a permutation of a chromatic pc set of order 3.
[31] In example 1.8, The CIG's are represented in their prime form (with pitch class number 0 (C) as their base). We will see that—since intervals are what counts in CIG's and not pc's—all CIG's may appear in any transposition.



Example 1.8: All 54 CIG's (chromatic interval groups).

1.1.4 Dissonance and amotivity

The minor second (ic 1) is the most frequent interval class between consecutive series notes in atonal series. The series of Webern's *String Quartet* op. 28, for instance, contains six minor seconds (see Example 1.11). And when two consecutive notes of the series are not ic 1 apart, the interval class between one of these notes and the next note in the series is necessarily ic 1. Ic 1 is therefore predominant in atonal series (consisting entirely of CIG's). Assuming that ic 1 is the most dissonant interval—the interval with the highest "degree of dissonance"[32], as Smith Brindle calls it—atonal series easily allow for the most dissonant sound combinations, and thus yield the most dissonant music. A compositional technique based on series that are constructed exclusively with CIG's therefore seems to be appropriate to accomplish my aesthetic goal of atonal and dissonant music.

The dodecaphonic technique—even when restricted to the use of atonal series—didn't seem to be sufficient for my purpose however, because it provides no solution to my concern about the justification of melodic intervals other than ic 1. By constructing an atonal dodecaphonic series, I would have to choose ten CIG's from the complete set of fifty-four possible CIG's. This raised the question: Why would I choose certain CIG's and leave out all the others? I felt the need for criteria to justify this choice. If, furthermore, I would use a particular CIG more than once in a series, its importance would increase. It would become a structural melodic motive in the series.

Grove's Dictionary defines a **melodic motive** as "the shortest subdivision of a [melodic] theme or phrase that still maintains its identity as an idea"[33]. It is the smallest group of notes that allows for

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[32] Reginald Smith Brindle, *Serial Composition*, p. 36.

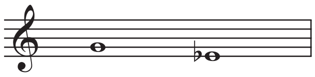
[33] William Drabkin, *Motif*, in *Grove Music online*, www.oxfordmusiconline.com+19221?q=motif&search=quick&source=omo_gmo&pos=1&_start=1#firsthit [last accessed: 11 March 2013]. Grove's Dictionary uses the spelling "motif". I prefer the alternative spelling "motive" which will exclusively be used in the present text, except as part of a quote.

recognition or identification of or within a piece. Echoing Schoenberg, Anton Webern calls a motive "the smallest independent particle of a musical idea"[34] that is recognized because it is repeated.

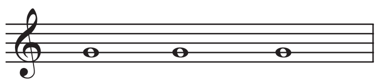
The smallest possible melodic motive consists of three pitch classes, at least two of which are instances of different pitch classes. A melodic cell that contains only two pitch classes is a melodic interval, which is too small and too common to characterize a piece. Grove's Dictionary states the opening theme of Beethoven's *Sonata in E* op. 109 as a case. In this example, a whole bar can be said to constitute a motive, rather than the recurring pattern of note pairs, "since [only] the two pairs of notes together form an identifiable contour; the two-note members might then be called 'cells'"[35], as shown in Example 1.9.

Example 1.9: Opening theme of Beethoven's *Sonata in E* op. 109
(source: Grove Music online).

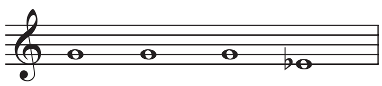
A melodic figure containing three times the same pitch class is not considered to be a motive either, since it lacks 'identity as an idea' (it is insufficient to allow for identification of a piece). In the example below, cells containing two pitch classes (Example 1.10 a) or three identical pitch classes only (Example 1.10 b) cannot be considered as characteristic for a piece. They are not motives. Example 1.10 c, on the other hand, shows a genuine melodic motive (containing four pitch classes, and two different ones, G and E flat) since the piece it is taken from can easily be identified by it.



Example 1.10 a: Interval.



Example 1.10 b: Three identical pc's in a melodic cell.



Example 1.10 c: Melodic motive from
Ludwig van Beethoven's *Fifth Symphony in c minor* op. 67.

If one would want to write strictly amotivic music, one would have to avoid the repetition of every possible melodic motive, even the smallest one consisting of three pitch classes that are not all identical. This would limit the number of notes a composition can contain to 1718,[36] not counting

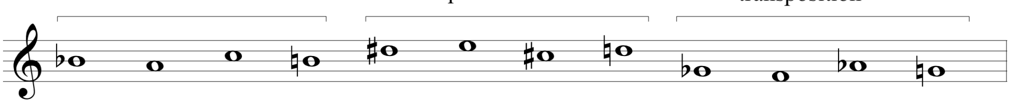
[34] Anton Webern, *Der Weg zur Neuen Musik*, re-edited by Willi Reich, Universal Edition, 1960, p.27, translated by Leo Black as: *The Path to the New Music,* Theodore Presser Company, 1963, p. 25, quoted in: Joseph N. Straus, *Remaking the Past: Musical Modernism and the Influence of Tonal Tradition*, Harvard University Press, 1990, p. 39, and in: Edward Campbell, *Boulez, Music and Pilosophy*, Cambridge University Press, 2010, p. 157.

[35] William Drabkin, *Motif*, in *Grove Music online*, www.oxfordmusiconline.com+19221?q=motif&search=quick&source=omo_gmo&pos=1&_start=1#firsthit [last accessed: 11 March 2013].

[36] Counting an ordered pc-set of order 3 with at least two different pc's as a motive, there are 1716 possible (smallest) motives. This is calculated as follows: Let {0,1,2,3,4,5,6,7,8,9,10,11} be the set of all pc's. It is divided in ordered sets of

notes immediately repeated more than twice (the melodic motive from Ludwig van Beethoven's *Fifth Symphony* in Example 1.10 c, for instance, counts for three notes only: two G's and one E flat). This is of course a purely theoretical consideration, because in practice a group of notes is neither perceived nor considered as a motive unless its repetition and its function as an 'identifiable contour' can be perceived, that is, if it 'maintains its identity as an idea'. Even in 'motivic' (or thematic) composition not every succession of three notes is perceived or considered as a melodic motive. Only those note groups that have been conceived and can be recognized as motives are considered as such.

I call motives that result from the way series are *structured* **structural motives**. Dodecaphonic series—even atonal ones—may contain structural motives. "Indeed in Webern's own typical use of the row in either three- or four-note self-referential partitions, the 'motive' is the substrate of the 'theme'"[37]. The 'theme' refers here to the undeniable "'thematic' […sense of the dodecaphonic approach…], characterized by an ordered series of intervals."[38] The atonal series of Webern's *String Quartet* op. 28, for instance, is based on the well-known BACH-motive. This melodic motive is followed by its transposed inversion and transposition, resulting in the motivic series in Example 1.11.



Example 1.11: Series of Anton Webern's *String Quartet* op. 28, a case of a motivic series.

An atonal dodecaphonic series can easily be made strictly amotivic. In order to do this, not even the smallest possible motive—a three-note CIG—should occur more than once in the series, as is the case in the series of Webern's *Symphonie* op. 21, but not in the series of his *String Quartet* op. 28. Transpositions of a CIG are also to be excluded. Notes 9 through 11 in Webern's *String Quartet* are a transposition of the CIG formed by notes 1 through 3. Therefore, the series is not amotivic.

One may object that strict amotivity is only possible if the three transformations (inversion, retrograde and retrograde-inversion) of each CIG (what George Perle calls the "basic cell")[39] are excluded. I take this restriction into consideration in the next paragraph.[40] Note also that although a series may be amotivic, the music that results from it may contain melodic motives, depending on the compositional approach of the composer. These motives may be structurally planned, but they may also be the result

order 3 containing at least two different pc's (hereafter called motives). There are 12 possibilities for the pc in the first position of the motives: [0 . .] through [11 . .] (the dots indicate open positions in the motives).
To fill in the second position of the motives, two distinct cases have to be considered:
a) For each of those possibilities, the second pc can be the same as the first only in one way: [0 0 .] through [11 11 .]
The third position can be filled in eleven ways for each of those motives (only the pc in position 1 and 2 is excluded).
In total, there are 12 x 11 = 132 possible motives.
b) Added to this are the motives in which the second pc is different from the first. This is possible in 12 x 11 = 132 ways (variations of 12 elements in 2 positions). In each of those motives, the third pc can be any of the 12 pc's. In total 12 x 11 x 12 = 1584 possible motives.
Adding the number of motives in a and b, there are 132 + 1584 = 1716 possible smallest melodic motives.
Since strictly speaking (and in theory only), any three successive notes in a piece (discarding notes repeated more than twice) can be considered to form a 'smallest motive', the number of notes in an amotivic piece (again not counting notes repeated more than twice) is limited to 1718 (notes 1 through 3 are the first smallest motive, note 2 through 4 the next, note 3 through 5 the next, and so on. The 1716th and last possible smallest motive contains notes 1716, 1717 and 1718 of the piece. A 1719th note would repeat a smallest motive between notes 1717, 1718 and 1719).

[37] Jonathan Dunsby, *Thematic and Motivic analysis,* in Thomas Christensen (ed.), *The Cambridge History of Western Music Theory*, Cambridge University Press, 2002, p. 910.

[38] Jonathan Dunsby, *Thematic and Motivic analysis,* p. 910.

[39] George Perle, *Serial Composition and Atonality, An Introduction to the Music of Schoenberg, Berg, and Webern,* University of Califormia Press, 6th revised edition, 1991, pp. 9-10..

[40] Contour transformation is also not taken into consideration, since it has no structural function in the construction of the series. On contour transformation see Edward Pearsall & John W. Schaffer, *Shape/Interval Contours and Their Ordered Transformations: A Motivic Approach to Twentieth-Century Music Analysis and Aural Skills,* in *College Music Symposium*, 45, 2005, pp. 57-80.

of contingency, much like clouds that may resemble existing objects to us without being conceived that way.

Amotivity of a dodecaphonic series alone doesn't solve the problem of the justification of melodic intervals. It provides no criterion of choice. It leaves the question why to choose ten particular CIG's and leave out all the others unanswered. By including every possible CIG in a series—which is then no longer a dodecaphonic series, of course—one not only avoids certain CIG's (the ones in the series) to be more important than others (the ones left out), but one avoids the problem of choice altogether. It also provides a justification to include the transformations of CIG's mentioned above.

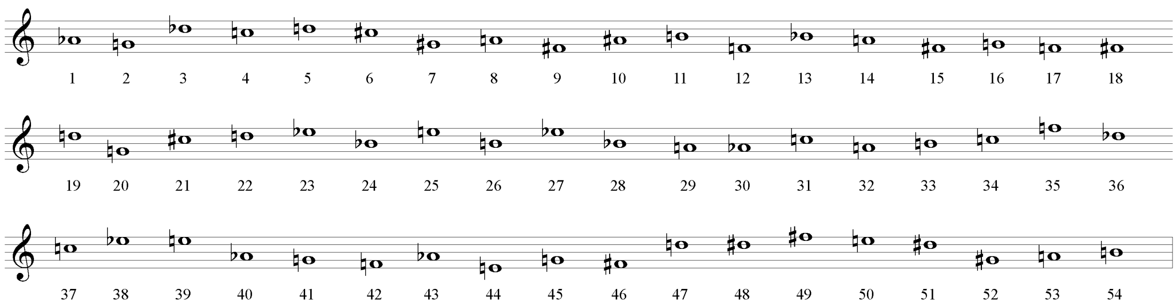
Finally, in order not to distinguish between the first and last CIG in the series on the one hand, and the other ones in between on the other, I determined that the series of my technique should be 'closed'. This means they have a ring structure, they bite their own tail, the last notes of the series form a CIG (one that does not occur elsewhere in the series) together with the first notes (notes 53, 54 with note 1, and note 54 with notes 1 and 2).

My search for a technique that allowed for an atonal, dissonant, and (structurally) amotivic idiom, and that at the same time solved the problem of melodic interval justification, thus resulted in 1997 in the construction of **54-CIG-series**, or **CIG-series** in short. I named the technique based on these series **chromatic interval group serialism**, or **CIG-serialism**.

The features of a CIG-series are:

- It consists exclusively of CIG's, regardless of transposition (making it atonal and dissonant).
- Every CIG occurs exactly once (making the series structurally amotivic).
- The series is 'closed'.

An example of a 54-CIG series is shown below (Example 1.12). It is the series of my piece *Comme un flocon de neige* for flute and ensemble (2007). As can be seen, notes 1 to 3 in this series constitute a CIG. Notes 2 to 4 are different CIG. Notes 3 to 5 another one that did not occur before, and so on until the end of the series (notes 52 to 54). Notes 53-54-1 and 54-1-2 constitute the two remaining CIG's that did not occur in the series before.



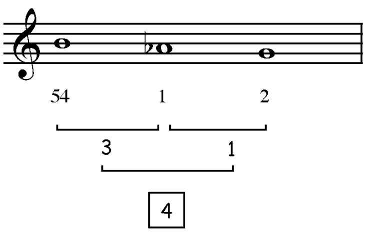
Example 1.12: Series of *Comme un flocon de neige*.

1.1.5 Constructing a rhythmic harmonic substructure

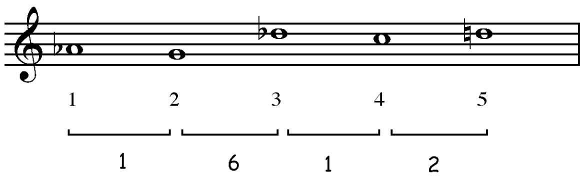
According to the assumptions made by Reginald Smith Brindle about the construction of atonal series, and assuming that ic 1 is the most dissonant interval, the CIG-series should be an adequate basis for my aspiration to compose atonal and dissonant music that allows for a justification of musical intervals different from ic 1. To preserve my personal idiom of non-tonal central pitch classes that characterised my compositions before 1997, I worked out a system that turns the series into a frame of rhythmic cells attached to every note of the series. I called this frame the **rhythmic-harmonic**

substructure (hereafter called **RHS**) of the piece. The repeated pitch classes within a rhythmic cell allow for central pitch classes. The RHS provides an intermediate step between series and score in the process of composition. The procedure of transformation from series to RHS is tailor-made for every piece, but always based on the interval class content of the series. The interval class content of a series is the number information provided by the interval classes between the pitch classes in the series.

Example 1.13 a below shows the unordered interval classes[41] between notes 54 and 1 (unordered ic 3), and between notes 1 and 2 (unordered ic 1) of the series of my piece *Comme un flocon de neige* for flute and ensemble (from 2007).[42] Example 1.13 b shows the unordered interval class content between the first five notes of the same series.



Example 1.13a: Notes 54-1-2 of *Comme un flocon de neige* with interval class content .



Example 1.13b: First five notes of *Comme un flocon de neige* with interval class content .

As a first step in the construction of a RHS, I determine the number of note lengths in the rhythmic cells attached to every note in the RHS. In the case of *Comme un flocon de neige*, this was done by adding the unordered interval class content of the intervals just before and just after the series note in question. Note 1 of the series of *Comme un flocon de neige* (A flat) is preceded[43] by note 54 (B) and followed by note 2 (G). The unordered interval class content of the interval between notes 54 and 1 (B and A flat) is three semitones, and between note 1 and 2 (A flat and G) one semitone.[44] Adding up interval class contents (3+1) results in 4, the number of note lengths for the rhythmic cell attached to note 1 in the series is therefore four. The same procedure is applied to the whole series (notes 2 and 3: seven note lengths, note 4: three note lengths, etc. as can be deduced from example 1.13b).

To determine the duration (note length) of every note in the rhythmic cell, as the second step in the construction of a RHS, other formulas based on the interval class content of the series are used. In the

[41] An ordered interval class is the number of semitones between two pitch classes, taking into account whether the interval class is ascending or descending, indicated by a + or - sign respectively (e.g. an ascending ic 1 has value +1, a descending ic 1 has value -1). An unordered interval class is the absolute value of the ordered interval class (e.g. 1 for both ascending and descending ic 1). Note that Joseph Straus doesn't distinguish between ordered and unordered interval classes, although he does so between ordered and unordered pitch intervals and pitch-class intervals (see: Joseph N. Strauss, *Introduction to Post-Tonal Theory*, Pearson Prentice Hall, 3rd edition, 2005, pp. 8-11).

[42] An extended example of how the RHS of *Le sourire infini des ondes* for ensemble was constructed will be given in the description of artistic output.

[43] Since an CIG-series is closed, it has no real first or last note. Any note in the series can be the first. Therefore the first note can be said to be preceded by the last note of the series.

[44] In the case of *Comme un flocon de neige,* unordered interval classes are used. In other pieces, such as *A l'image du monde…originel* (see Part 3), the direction of (ordered) interval classes is relevant.

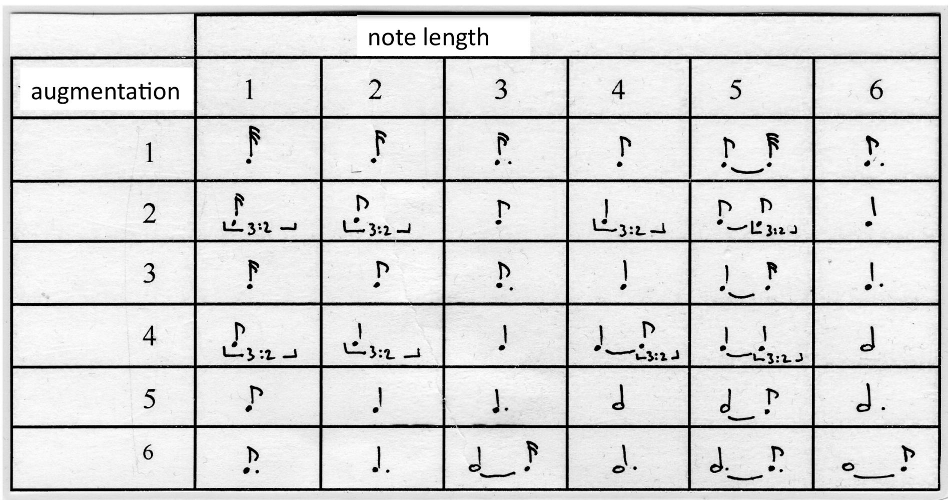
case of *Comme un flocon de neige,* the durations were determined as follows: for note 1—which has a rhythmic cell with four notes, as we have seen—the first four values in the series of unordered interval class content starting at the interval to the right of note 1 are used (see example 1.13b). Those values are:

1 6 1 2

The values for note 2 start from the next value in the series (the interval class between note 5 and 6) and continue to the right, etc. When the end of the series of ordered interval classes is reached, the series starts over from the beginning. Again, as is the case for the series, there is no real beginning or end to the series of interval class content.

To attach note lengths to the strings of numerical values thus obtained, first a 'length unit' or 'augmentation' is determined for every note of the series. This **augmentation** is the note length corresponding to note length value 1, the length unit, varying between demi-semiquaver and dotted quaver, as indicated in the column under "1" in the rhythm chart below (Example 1.14). I could change this rhythm chart for every piece (adding quintuplets for instance), but to date I did not feel the need for this change, so I used the same rhythmic chart for all my pieces since 1997, with the exception of the first piece written with the CIG-technique (*Les racines du monde* for piano (1998)).

rhythm chart



Example 1.14: Rhythm chart used to determine note lengths in the rhythmic cells of a RHS.

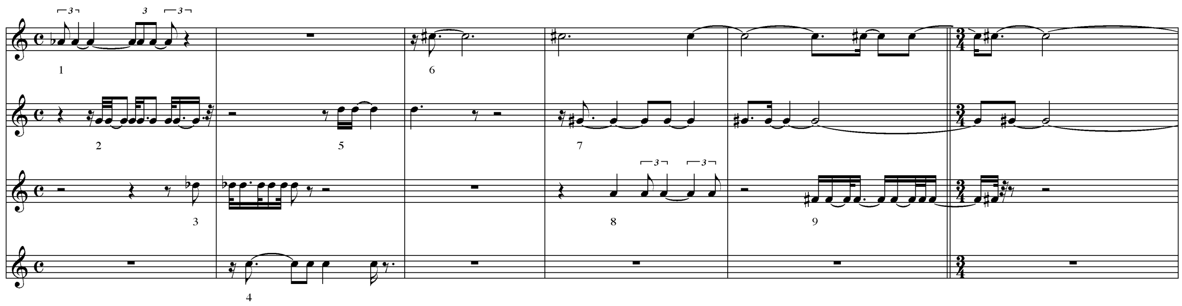
In the case of *Comme un flocon de neige*[45], the value used for augmentations is the same as the value for the number of notes in the rhythmic cell (modulo 6)[46]. The rhythmic cell of note 1 has therefore augmentation 4 (3 + 1); note 2 and 3 have augmentation 1 (7 = 1 (mod6)), etc.

Knowing that the rhythmic cell of note 1 contains four note lengths (1, 6, 1 and 2) in augmentation 4 (1= triplet quaver, 2 = triplet crotchet, etc.), the rhythmic cell for note 1 is as shown in Example 1.15.

[45] For other pieces, other formulas may be applied.
[46] Since there are only six augmentations, values higher than 6 are "reduced" to lower values modulo 6 (7 becomes 1, 8 becomes 2, …, 13 is again "reduced" to 1, 14 to 2, …).

Example 1.15: Rhythmic cell for note 1 of the series of *Comme un flocon de neige.*

After determining rhythmic cells for all series notes, the cells are placed in a metric frame, the RHS of the piece. This can also be done according to strict formulas (as is done in *Es träumte mir...* for for 6-part male choir (from 1998), in the string trio *Etoiles peintes* (2000) and in other pieces)[47], but in the case of *Comme un flocon de neige, Les racines du monde,* and several other of my compositions, this was done freely, aiming at a balanced rhythmic structure. The first six bars of the RHS of *Comme un flocon de neige* are shown in Example 1.16.



Example 1.16: Bar 1-6 of the RHS of *Comme un flocon de neige*.

1.1.6 Turning the rhythmic-harmonic substructure into a surface structure

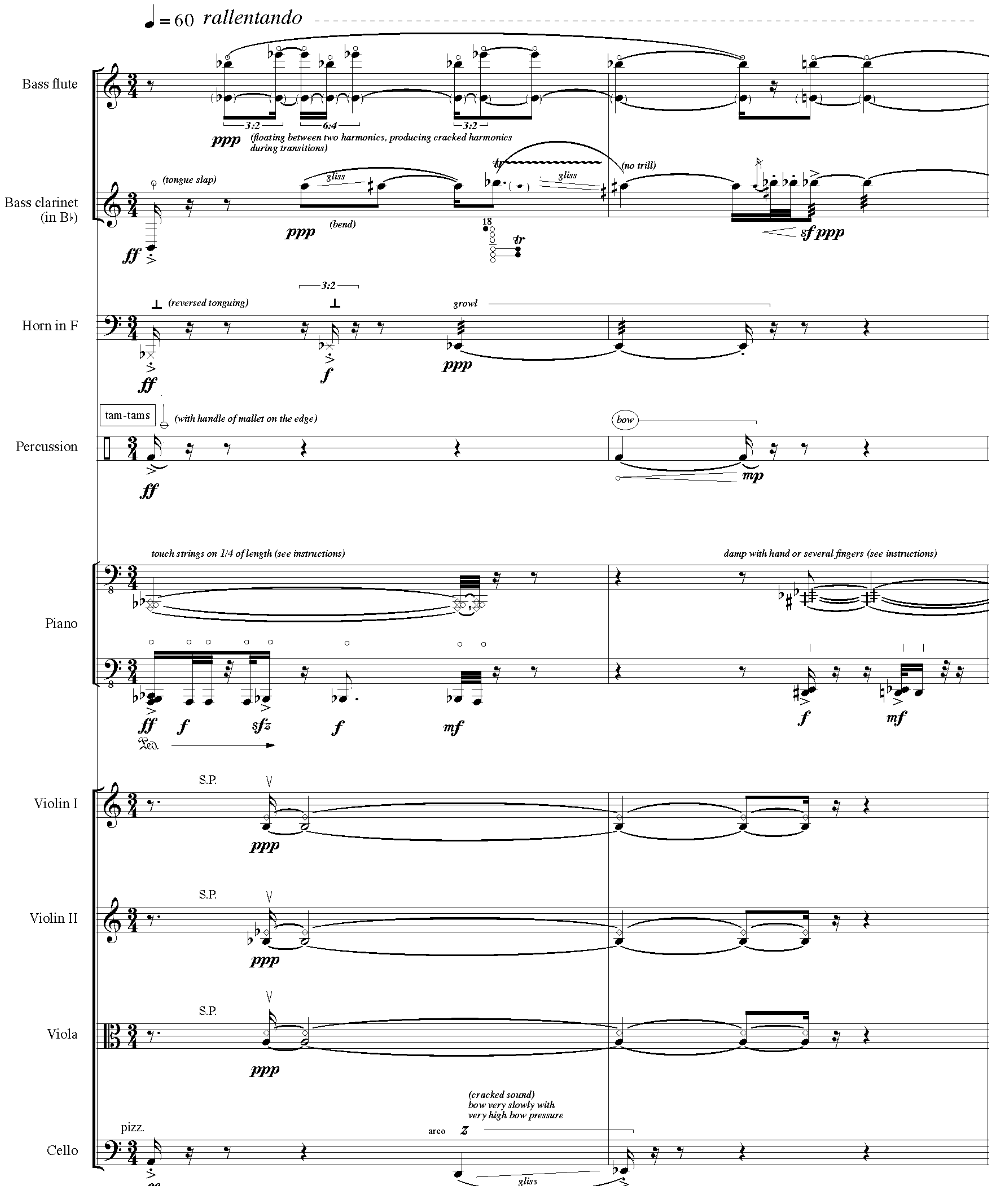
Once the RHS of a piece has been determined, the composition of the final score can begin, turning the RHS into the surface structure of the piece. In this process, artistic creativity prevails over structural strictness and rigor. Still this process is performed according to some well-determined rules. Every note occuring in the RHS should also occur in the score at the corresponding place in the RHS. The first note of *Comme un flocon de neige*, for instance, should be an A flat, since it is the first note in the RHS. The instrumentation, pitch, instrumental technique, etc. used are usually under the composer's control however—but not necessarily, as in the case of the cantata *Close my willing eyes* for three sopranos and ensemble (from 1999) .

Example 1.17 a shows the first two bars of the RHS of *Le sourire infini des ondes* for ensemble (2009), a piece that will be discussed in greater detail in the description of artistic output (see Part 3). The RHS contains three transpositions of the same form of the series (I, II and III) a semi-tone apart, with different rhythmic cells attached to them. Example 1.17 b shows the score generated from the RHS for the same two bars.

The RHS starts with three notes: B, B flat and A. Those pitch classes also occur on the first beat of the score (B is played by the bass clarinet, B flat by the horn, and A by the cello, and all three pitch classes occur in the piano part).

[47] For an example of systematic distribution of rhythmic cells in the RHS, see the analysis of *Le sourire infini des ondes* in Part 3.

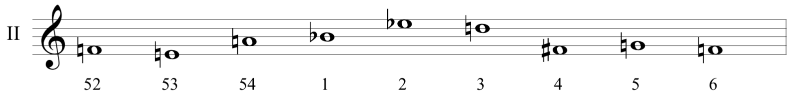
Example 1.17 a: First two bars of the RHS of *Le sourire infini des ondes* for ensemble.



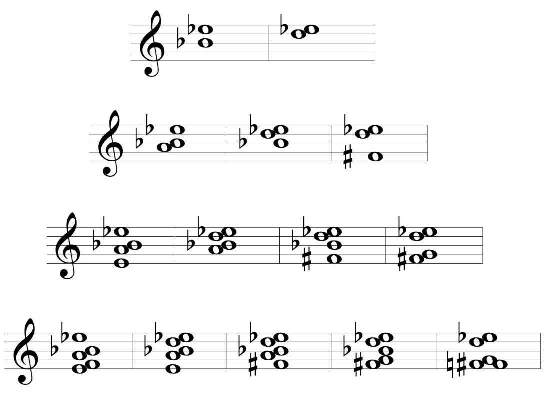
Example 1.17 b: First two bars of the score (in C) of *Le sourire infini des ondes* for ensemble.

An additional **'chord rule'** is used to construct chords on any series note at any given moment in a piece. The rule states that series notes in a RHS can be accompanied by the series note that immediately precedes and/or follows it (the neighbouring notes in the series). In some pieces this rule is extended: if a neighbouring note occurs in the chord, the next or previous note in the series may also be used, and so on. This way, chords are build that consist entirely of notes belonging to the CIG's, thus preserving the alleged high degrees of atonality and creating dissonance. The range of extension of the chords varies from piece to piece or from section to section within a piece according to the desired complexity of texture. Chord notes can also be used as grace notes preceding the RHS notes. Abundant grace notes have always been part of my style and idiom.

Examples 1.17 a and b above and 1.18 a and b below illustrate the procedure of chord formation according to the chord rule. The third beat of the RHS of *Le sourire infini des ondes* starts with B flat and E flat. In the score, the bass flute and piano play B flat and the horn plays E flat on this third beat. But there is also a D in the cello, which does not occur in the RHS at that moment. This D is the third note in version II of the series (the one starting on B flat) in the RHS, and a neighbouring note of E flat in the series (the note following E flat). The addition of D turns the consonant interval B flat – E flat in the RHS into a (dissonant) CIG of order 3. Example 1.18 a shows the eight notes surrounding note 2 (E flat) in version II of the series of *Le sourire infini des ondes*. Example 1.18 b shows all the possible chords that can be constructed on this note containing two to five pitch classes based on the chord rule described above. Whether a series note is extended into a chord, and which chord, is determined by me. This way I, as a composer, have extra control over the texture and dissonance of the piece at all times.

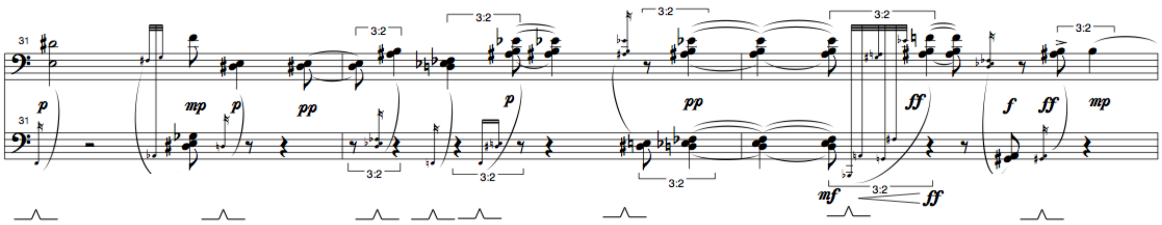


Example 1.18 a: Eight notes surrounding note 2 (E flat)
in the second transposition of the series of *Le sourire infini des ondes.*



Example 1.18 b: Possible chords up to five pc's on note 2
in the second transposition of the series of *Le sourire infini des ondes.*

If we compare the excerpt from *Les racines du monde* (1997) in Example 1.19 below with the excerpt from *Monodie* in Example 1.1, it is clear that, although the pieces are written with a different technique, they belong to a similar sound idiom characterized by its ornamented central tones, but, whereas *Monodie* was completely composed around pitch class A, there is more pitch variation in *Les racines du monde*, as was the goal of the development of CIG-serialism.



Example 1.19: Bar 31-33 of *Les racines du monde* for piano solo.

As a final remark, a parallel may be drawn between the levels of series, rhythmic-harmonic substructure, and score in my CIG-serial technique on the one hand and Schenker's idea of strata[48] or structural levels[49] on the other. The series would then correspond to Schenker's "background" the rhythmic-harmonic substructure to his "middleground" and the score to the idea of "foreground"[50].

1.2 Teleology and mutations

As we have seen, the series lies at the basis of the construction of the RHS, which, in turn, provides material for the score. The processes involved happen, as was discussed, according to strictly determined rules as well as on the basis of free artistic choice and intuition. In both cases the series contains the basic information for the construction of my pieces. It can therefore be compared to the genetic material of living organisms, which contains the necessary information for the 'construction' of the living organisms.[51] The series provides the 'genotype' of a piece; the score is the 'phenotype'. Strict transformative formulas in combination with artistic choice rule the compositional processes of CIG-serialism; physico-chemical laws determine the processes involved in biological processes.

There is an important difference between the processes of CIG-serial composition and genetic biology however. Contrary to biologic evolution, CIG-serialism is a teleological process. This means that every step in the process can be influenced by the envisaged goal. Even before the construction of series, I have an idea in mind of how the piece is going to sound (this sound idea is usually the initialising step in the composition process). The construction of the series can therefore depend on the envisaged result. At all moments during the compositional process, there is a possibility of feedback between the construction of series, RHS and score, in order to obtain the envisaged sonic and narrative result. The envisaged result may influence the preceding steps in the process. The envisaged RHS may influence the construction of the series or the formulas for the transformation of the series into the RHS; the envisaged surface structure influences the construction of series and RHS and the transformational rules. The arrows in Example 1.20 show the different processes of CIG-serialism. The ascending arrows indicated the feedback processes.

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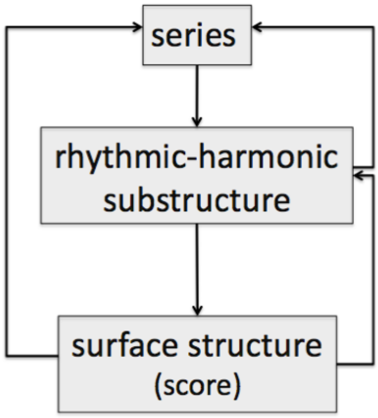
48 See Oswald Jonas (ed.) in Heinrich Schenker, *Harmony*, Elisabeth Mann Borgese (translator), The University of Chicago Press, 1954, p. XXII.

49 See: Allen Forte & Steven E. Gilbert, *Introduction to Schenkerian Analysis*, W.W. Norton & Cie., 1982, p. 131.

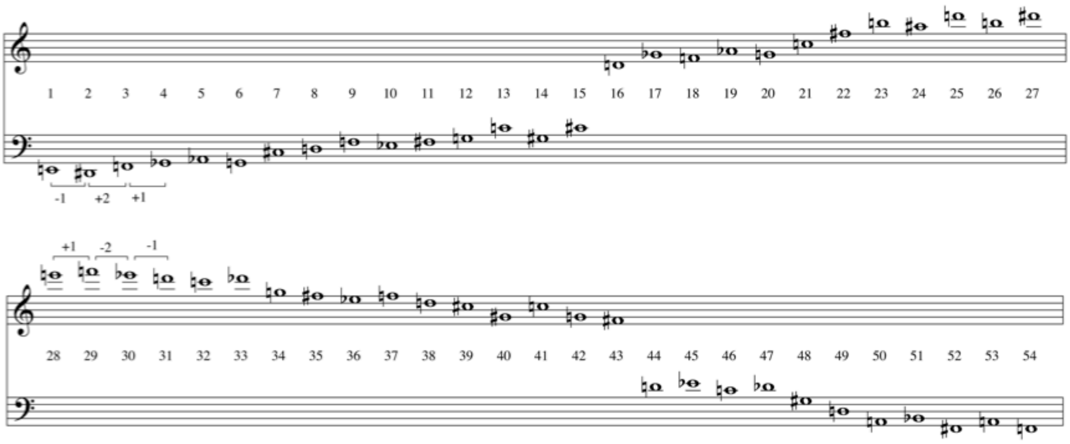
50 The background (*Hintergrund*) constitutes "the contents of a tonal work at the most basic level [represented by the *Ursatz* that] gives rise to more elaborate harmonic-contrapuntal designs. These in turn generate further development, in stages, until the final elaboration is reached, which is the piece itself with all its details of rhythm and tempo, dynamics and articulation, and scoring. This level is called the *foreground* of a composition (*Vordergrund*). Between the extremities of background and foreground lies the *middleground* (*Mittelgrund*), an area whose scope and complexity is dependent on the size and nature of the composition" (Thomas Christensen (ed.), *The Cambridge History of Western Music Theory,* Cambridge University Press, 2002, p. 819).

51 A similar comparison is used by Andrew Mead in his discussion of Milton Babbitt's music (see: Andrew Mead, *An Introduction to the Music of Milton Babbitt*, Princeton University Press, 1994, p. 7).



Example 1.20: Transformational processes in CIG-serialism.

Let us have a look at the construction of the series of *Après la pluie* for piano and live electronics (2008), as an example of these feedback processes. In this piece, the sound of the piano is manipulated by live electronics. The level of manipulation evolves during the nine sections of the piece, gradually increasing towards section 5, and then decreasing until almost no manipulation is performed in the final section (section 9). In order to obtain numerical values that make this arc shape of manipulations possible, I constructed a series that has an arc shape, in which all net rising CIG's occur in the first half and net descending intervals in the second half. A net rising CIG is a CIG in which the sum of the two ordered interval classes it contains is positive. The first CIG in the series, for instance, constitutes of ordered ic -1 an +2 (see Example 1.21). The sum of these ordered interval classes is +1, a positive number. The first CIG in the series is therefore a net rising CIG. The second half of the series is constructed as the inversion of the first half, which automatically leads to the desired result (the arc shape of the series) as can be seen in Example 1.21. Also, the smallest numerical values occur at the beginning of the series, gradually increasing towards the end of the first half of the series (note 27).



Example 1.21: Series of *Après la pluie* for piano and live electronics.

At all times during the compositional process, I can decide whether I want to abide strictly by the self imposed rules or make free adaptations. Since I look upon the series as the genetic material of my compositions[52], I call the free (sometimes accidental but mostly deliberate) deviations from the rules **mutations**. In the biological processes of construction of new genetic material—the recombination of existing material in sexual reproduction—as well as in its transformation, random mistakes occur

[52] Arnold Schoenberg too "liked to call [the relationship of material at all levels] 'subcutaneous'—that is, more than skin-deep—but in fact it is positively molecular. The note-row is, so to speak, the DNA molecule of twelve-tone music: the agent which stamps every bar, every theme, every chord, as belonging to a single, unique work." (Malcolm MacDonald, *Schoenberg*, Oxford University Press, 2008, p. 142).

(which are the driving power behind the process of evolution). In the processes of my compositional practice, mutations can appear at all levels: in the implementation of the series, in the construction of the RHS, as well as in the composition of the surface structure (the score). They are mostly the result of artistic considerations, not mere chance (random mistakes), as is the case in biological mutations[53]. In this respect artistic creation differs from biology: artistic mutations can be teleological.

The example below shows a mutation in *Les racines du monde* for piano solo at the level of RHS construction. In this piece, the position of every rhythmic cell in the RHS was not determined by a formula, but was the result of my creative search for rhythmic balance. Whenever it seemed appropriate, I changed the order of rhythmic cells in the RHS; this implies that a rhythmic cell of a series note can start before the rhythmic cell of the previous series note in the RHS. Example 1.22 a shows the first nine notes of the series of *Les racines du monde*. Example 1.22 b shows the opening bars of the score of that piece. As can clearly be seen, the first rhythmic cell occuring in the score is the cell of the second series note (B flat in the right hand part). The rhythmic cell of note 1 (A) only starts in the second bar of the piece (left hand part).

Example 1.22 a: First nine notes of the series of *Les racines du monde* for piano solo.

Example 1.22 b: First bars of *Les racines du monde* for piano solo.

As a restriction to this mutational process, I allowed swaps of rhythmic cell introduction only if the series note of the postponed rhythmic cell is played as a grace note for the first note of the anticipated rhythmic cell. In Example 1.22 b, for instance, the first B flat of bar 1 is preceded by a grace note A (the series note that is anticipated by the rhythmic cell on B flat). This way, although the order of rhythmic cells is not preserved, the series notes themselves are still introduced in the 'right' order.

At other times, deviations from the structure yielded by the RHS are introduced if the sounding result can be improved by doing so (according to my personal aesthetic judgment as a composer). For instance, there was a gap of more than two crotchet beats between bar 148 and 149 in the score of *Les racines du monde*, because of the way the RHS was constructed (see Example 1.23 a). This resulted in an interruption of the building up of the climax. To avoid this shortcoming, I left out the two beats of rests, as is shown in example 1.23 b, because I judged that leaving out those beats would produce a better result. I claim that, at all times, my personal aesthetic judgment is more important than strictness of rules in the process of composition.

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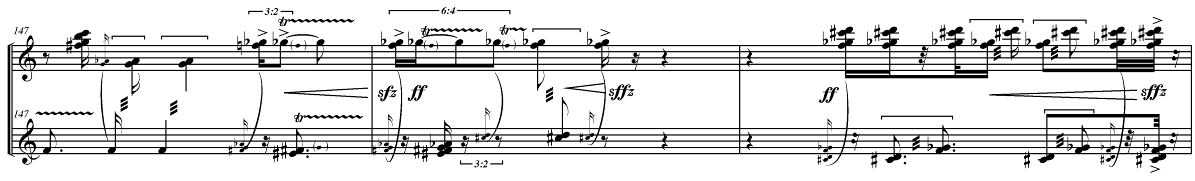
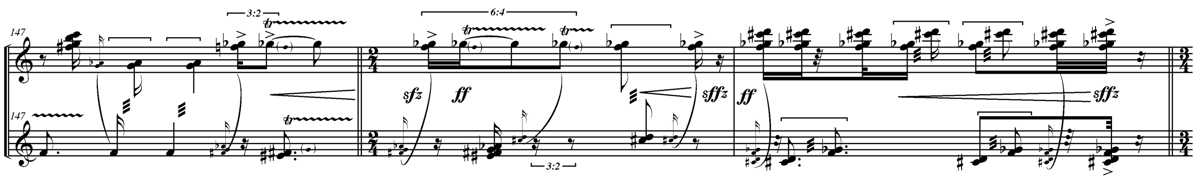
[53] I restrict the meaning of 'biological mutations' to non-artificial biological mutations in the present context. I am leaving artificial mutations (as they are produced in genetic manipulation) out of the picture here, although there are striking similarities between the processes of 'genetic' control in my approach to CIG-serialism and genetic manipulation.



Example 1.23 a: Bar 147-149 of *Les racines du monde* for piano solo, before mutation.



Example 1.23 b: Bar 147-149 of *Les racines du monde* for piano solo, after mutation.

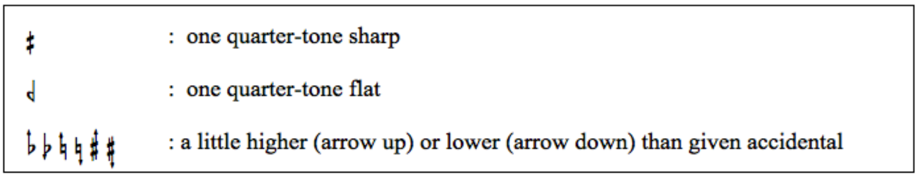
The distinction between deliberate (conscious) and unintentional (accidental) mutations is not always clear. The latter could be called 'mistakes', but I see them rather as results of the composition process that have the same artistic legitimacy as conscious mutations. Such mistakes, or rather unintentional mutations, may occur at all times, including during the transcription of the manuscript to the computer version of the score. As long as they do not result in theoretical or physical impossibilities, it is exclusively up to my aesthetic judgment whether I correct the mutations or not. Most of the time, minor deviations in the surface structure due to unintentional mistakes have no relevant impact on the sounding result. As long as the surface structure is in accordance with the way I mentally hear the piece, there is no reason to make any changes. Imperfections due to unintentional mutations may even add to the aesthetic value of a piece, as is the case with the *campanile* of Pisa, better known as *the Leaning tower*. Arnold Schoenberg too, was very clear about deviations from technical rules in his music (which may arguably have been unintentional), as the following anecdote related by Eugen Lehner, violist of the Kolisch Quartet, demonstrates:

> Once, we spent a summer with Steuermann and Kolisch, down in the mountains in a village, and we analyzed the whole third quartet, bar by bar, note by note and, in those nearly thousand bars, to our great satisfaction, we found two places where Schoenberg had made a grave mistake. So, as soon as we came to Berlin, the first time we went to Schoenberg, we showed it to him. "Is that a misprint?" "No, no, that's correct." So we said, "Oh, it's not a misprint, then it's a mistake." Then we explained to him and Schoenberg got mad—red in the face! "If I hear an F-sharp, I will write an F-sharp; if I hear an F-natural, I will write an F-natural. Just because of your stupid theory, are you telling me what I should write?"[54]

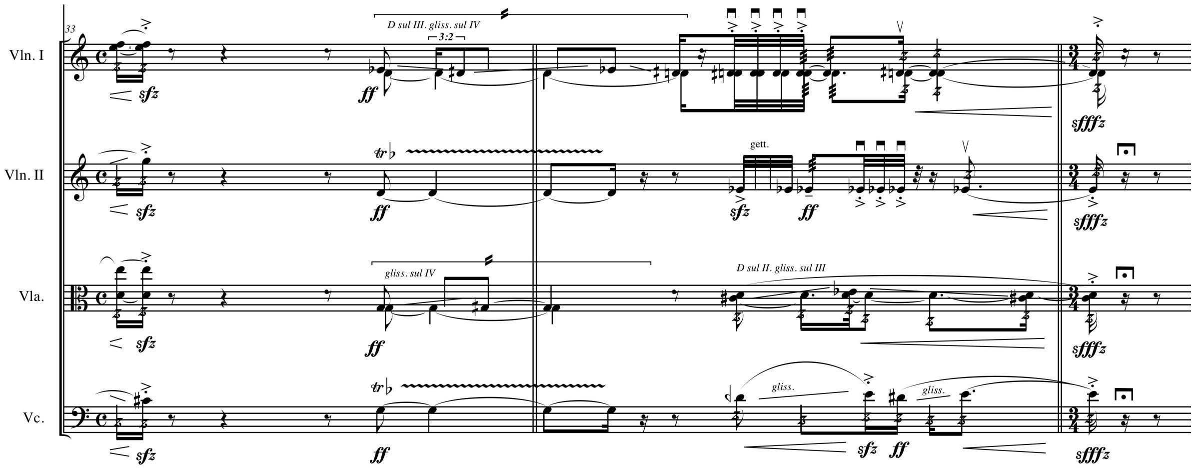
1.3 A note on micro-intervals

The technique of CIG-serialism is based on the use of the twelve pitch classes of the chromatic scale. It does not consider micro-intervals. Micro-intervals (quarter-tone sharps and flats and others such as listed in Example 1.24 below) do occur in my music however, yet they never have any structural function (they are never part of the RHS of a piece). Micro-intervals only have an ornamental or colouring function in my music. They always accompany structural notes the way semi-tone intervals ornament the central notes in my works prior to 1997, or they fill in semi-tone intervals, as can be seen in example 1.25 below, an excerpt from my piece *Le sourire infini des ondes* for ensemble (2009).

[54] Eugen Lehner, quoted in Joan Allen Smith (ed.), *Schoenberg's Way*, in Perspectives of New Music, Vol. 18, N°1/2 (Autumn, 1979 - Summer, 1980), p. 263.

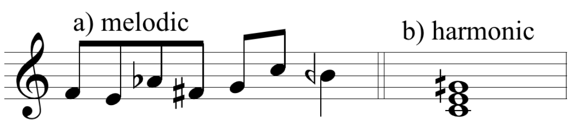


Example 1.24: Notation of micro-intervals in my music.



Example 1.25: Micro-intervals in *Le sourire infini des ondes* (bar 33-34).

Structural micro-intervals in an idiom that is based on the pitch classes of the chromatic scale (or even more so, on the pitch classes of a diatonic scale) in well-tempered tuning may easily be perceived as being ‘out of tune’ by listeners who are acquainted with well-tempered tuning. The B quarter-tone flat in the highly chromatic melody of Example 1.26 a may be perceived as being either a flat B natural or a sharp B flat (or A sharp). The G quarter-tone sharp in the chord of Example 1.26 b may similarly be perceived as being sharp (tonally acculturated listeners will probably not interpret the note as a flat A flat). In a ‘non-chromatic’ context, this perception disappears if the scales, tuning, or sound combinations used deviate enough from the chromatic well-tempered scale, when melody and harmony occur in a context predominantly based on intervals that are different from the well-tempered chromatic intervals, as is illustrated in Example 1.27 a. The use of micro-intervals *in micro-interval combination* with ‘in-tune’ chromatic intervals in my music is an instance of this approach. In Example 1.27 b, listeners do not usually interpret the quarter-tone sharps as ‘out of tune substitutions’ for notes belonging to the chromatic scale, as was the case in Example 1.26 b, since the context makes it hard or impossible to determined which note is a substitution for which other ‘in tune’ note.



Example 1.26: The use of micro-intervals in a chromatic or diatonic context
as (a) melodic and (b) harmonic intervals.

Example 1.27: The use of micro-intervals in a non-chromatic micro-tonal context as (a) melodic and (b) harmonic intervals.

Extending CIG-serialism to the quarter-tone-chromatic scale with 24 (equal or unequal) quarter-tone intervals, or a scale based on another division of the octave, may be an interesting enterprise, but it would not be an option in a quest for atonal music. The concept of atonality used in the present dissertation is, as will be seen, strictly related to the use of diatonic and chromatic scales. Music that is structurally based on other types of octave division would be neither tonal nor atonal, but non-tonal.

Chapter 2. Restatement of the research questions

The construction of CIG-serialism as a means to obtain an atonal and dissonant musical idiom was based on two assumptions: Reginald Smith Brindle's claim about the construction of what he calls 'atonal series', and on the assumption that ic 1 is the most dissonant interval. Therefore the claim that CIG-serialism can yield highly dissonant and atonal music only stands if these assumptions can be confirmed, if it is true that ic 1 is the most dissonant harmonic interval, and if it is true that Reginald Smith Brindle's atonal series maintain "throughout the same degree of atonality"[55]. If an interval is said to be 'the most dissonant', or to have 'the highest degree of dissonance', how should we understand the idea of 'degree of dissonance (or consonance)'? And how should we understand Smith Brindle's idea of 'degree of atonality (or tonality)'? The phrasing of those ideas suggests that consonance and tonality can be quantified, that it is possible to claim that some intervals are more or less consonant than others, that music can be more or less tonal.

Therefore, before the research question, whether CIG-serialism results in highly dissonant and atonal music, can be answered, some questions about the concepts of tonality and consonance have to be answered: Can consonance and tonality be quantified? How do the concepts have to be (re-)defined in order to make quantification possible? How, if possible, can tonality and consonance be quantified? Is this quantification in accordance with commonly accepted ideas of tonality and consonance? It is only possible to speak of 'degree of consonance' and 'degree of tonality' after this quantification. Subsequently, the technique of CIG-serialism will have to be assessed according to this quantification, addressing such questions as: Are CIG-series highly atonal and dissonant? And if so, can the technique be adapted in order to result in even lower degrees of tonality and consonance? The search for answers to these research questions constitutes the structure of Chapters 3 to 5. In Chapter 3, tonality will be assessed and quantified; the same procedure will be applied to the concept of consonance in Chapter 4. The results will then be used to assess the CIG-technique and adaptations will be made if necessary and possible. This will be done in Chapter 5, the final chapter of Part 1.

The highly theoretical questions about tonality, consonance and technique raise further aesthetic research questions such as: "what is the link between the developed theory and my artistic practice?" or "how is the theory reflected in the resulting compositions?" These additional research questions are addressed in Part 2 (Chapters 6 and 7). Part 3, finally, will be the synthesis of theoretical and aesthetic consideration; it consists of the description of the artistic output of the research.

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[55] Reginald Smith Brindle, *Serial Composition*, Oxford University Press, 1966, p. 12.

Chapter 3. Tonality and atonality

3.1 Introduction and preliminary remarks

Quantifying tonality or atonality is only possible after clearly defining the concepts of 'tonality' and 'atonality'. Although (and maybe because) tonality is one of the most common concepts in the tradition of Western music, its definition leads to a high amount of ambiguity and controversy, as will become clear in what follows. Trying to define the concept of tonality is therefore considered to be a very ambitious enterprise. Indeed, tonality is a concept that has obtained a wide range of meanings over the centuries. "As a music-theoretical term, 'tonality' was first used by Alexandre Choron in 1810 […]"[56]. Ever since, new connotations have been added to its meaning and different definitions have resulted from this.

All definitions are to a certain extent pragmatic (and even arbitrary). They depend on the purpose they are meant to serve. The definition of tonality within an atonal context will therefore have to be different from the one that is used when one compares, for instance, seventeenth century music with modal music, but this is not a problem since there is, as will be seen, no single definition of tonality that is generally accepted and applicable to all contexts. Still it is desirable to define tonality in such a way that it could apply to the broadest possible field of investigation. To achieve this, a critical overview of the defining aspects of the most commonly accepted historical and contemporary concepts of tonality is necessary before even trying to attempt 'redefining' tonality. The majority of those customary definitions can be grouped in five main categories according to the central aspect they are based upon: the contrast with modality, the presence of a tonic, (harmonic) functionality and hierarchy, perception, and what will be called 'diatonicity'. Many of the discussed categories of definition are overlapping. It is for instance impossible to talk about the presence of a tonic without including its hierarchic and functional features. In what follows, each of the defining categories will be discussed separately and the concept of atonality will be defined accordingly. The description of traditional 'definitions' (or descriptions) of tonality will at times be rather sketchy. It is not my ambition to provide a comprehensive overview of the concepts in the present dissertation. All definitions will be explored exclusively to shed light on the general principles underlying the definitions in the light of the present purpose. This will then be followed by the introduction of the concept of 'degree of tonality' that allows for quantification, and, finally, the construction of a formula to calculate the degree of tonality of any pitch class set will be explained.

3.2 Customary definitions of tonality

3.2.1 Tonality and modality

A first category of definitions of tonality is based on an idea of tonality that does not take into consideration the advent of atonality at the beginning of the twentieth century, and only focuses on the contrast between the 'modal' idiom used in the Middle Ages and the Renaissance up to the end of the sixteenth century on the one hand, and the 'tonal' idiom of the era of common-practice between

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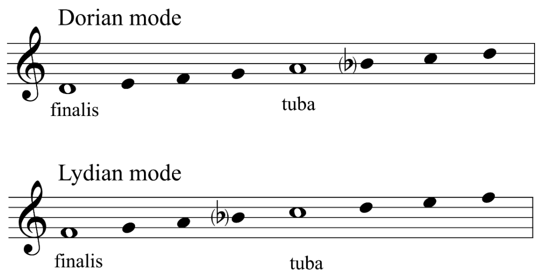
56 Brian Hyer, *Tonality*, in Thomas Christensen (ed.), *The Cambridge History of Western Music Theory*, Cambridge University Press, 2002, p. 726, referring to Alexandre Choron, *Dictionnaire historique des Musiciens Artistes et Amateurs, précédé d'un Sommaire de l'Histoire de la Musique,* Paris: Valade & Lenormant, 1810.

roughly 1600 and 1900.[57] Those were the only two recognized idioms in Western Cultural music until the end of the nineteenth century.

Medieval and Renaissance music uses the eight so-called medieval modes (hence the name 'modal music'), each with their own ending note (*finalis*) and reciting tone (*tuba*). By the eleventh century two of those medieval modes had become predominant: the Dorian mode and the Lydian mode. To 'soften' progressions between the *finalis* (F) and the fourth note (B) of the Lydian mode, the B was usually replaced by B-flat (see Example 3.1). "The resulting "Lydian" octave species […] is identical to what we would call the major scale"[58]. Other chromatic alterations "mainly affected the Dorian mode, the one closest to our minor mode. For example, there was a rule that a single B between two A's had to be a B-flat."[59] This 'adjusted' Dorian mode is identical to the natural minor scale (see Example 3.1).



Example 3.1: 'Adjusted' Dorian and Lydian modes.

Not only did the (adjusted) Lydian and Dorian modes become predominant in the eleventh century, the two modes were also used according to the character of the melodies the composers wanted to express: the Dorian mode for "penitent" melodies and the Lydian for "exultant" ones.[60] This habit evolved into the use of major scales for 'cheerful' music and minor scales for music with a 'sad' character. Thus the tonal idiom has its origin in the modal system; "the diatonic scale developed out of the church modes"[61], as Anton Webern wrote; the former can be interpreted as a historically evolved 'special case' of the latter, and both systems are therefore essentially of the same nature. Choron calls both of them tonal; he "contrasts *tonalité moderne* with *tonalité ecclésiastique*"[62]. This is in line with the view of Arnold Schoenberg, who "interpreted the modes as forms of major and minor"[63].

In what follows, no distinction will be made between Choron's *tonalité moderne* and *tonalité ecclésiastique*. The term tonality will include modality and will refer "to music based on the eight modes of Western Church as well as the major-minor complexes of common-practice music"[64].

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[57] Some authors limit the concept of 'common-practice' to the eighteenth and nineteenth centuries (see, for instance, Joseph N. Straus, *Remaking the Past: Musical Modernism and the Influence of Tonal Tradition*, Harvard University Press, 1990, p. 1).
[58] Richard Taruskin, *The Oxford History of Western Music, Volume 1: Music from the Earliest Notations to the Sixteenth Century,* Oxford University Press, 2010, p. 97.
[59] Richard Taruskin, *The Oxford History of Western Music, Volume 1,* p. 274.
[60] See: Richard Taruskin, *The Oxford History of Western Music, Volume 1*, p. 96.
[61] Anton Webern, *Der Weg zur Neuen Musik*, re-edited by Willi Reich, Universal Edition, 1960, p. 25, translated by Leo Black as: *The Path to the New Music,* Theodore Presser Company, 1963, p. 23.
[62] Brian Hyer, *Tonality*, footnote p. 726. Charles Rosen too includes modal music and defines (the era of) tonality as "a historical system that lasted from about 1500 to 1900" (Charles Rosen, *Arnold Schoenberg*, University of Chicago Press, 1975, p. 61).
[63] Clara Steuermann, quoted in Joan Allen Smith (ed.), *Schoenberg's Way*, in Perspectives of New Music, Vol. 18, N°. 1/2 (Autumn, 1979 - Summer, 1980), p. 270.
[64] Brian Hyer, *Tonality*, footnote p. 728.

3.2.2 The tonic

A second category of definitions of tonality focuses on the concept of 'tonic'. Definitions of this category are of the form: "In Western music, [tonality is] the organized relationship of tones with reference to a *definite center, the tonic*, and generally to a community of pitch classes, called a scale, of which the tonic is the *principle tone*"[65], or: "Tonal phenomena are musical phenomena […] arranged or understood in relation to a *referential tonic*."[66] Oxford English Dictionary defines Tonality as "[t]he principle or practice of organizing musical composition around a key note or tonic." To make those definitions clear, it is necessary to define the concept of tonic. What is a tonic?

The tonic is referred to as being the principal or referential[67], hence most important note in a piece or section of a piece, but it is not immediately clear what this means. What makes the tonic the most important note in a piece of music? Certainly it is not the fact that it occurs most frequently in a piece. Research by Bret Aarden on the scale-degree distribution based on a large sample (more than 65.000 notes) of conventional Western tonal melodies has shown that both in major and minor keys it is not the tonic but the dominant (fifth scale degree) that has the highest event frequency.[68]

Some theorists, among them Jean-Philippe Rameau[69] and Heinrich Schenker[70], base the importance of the tonic on the "natural superiority"[71] of the perfect fifth interval and the circle of fifths. "The major scale" they claim, "is simply taken to be the expression of the (tonal) system's root tones which have obeyed the laws of fifth evolution."[72] The problem here, however, is that the fourth scale degree (F in the key of C major) does not occur in the circle of fifths when the tonic is taken as a starting point. The circle of fifths starting on C is:

C G D A E B F# …

Therefore, if the circle of fifths were the natural basis for the superiority of the tonic, one would expect C major to contain F sharp instead of F natural. This is the case in the Lydian church mode on C, yet never is this argument used to claim the 'natural superiority' of the Lydian mode or its *finalis*; or it would account for the 'natural' hierarchically fundamental superiority of the subdominant (the fourth scale degree) instead of the tonic, since C is the subdominant of the major key with F-sharp (G major), which again has never been claimed, and rightly so.

Deduction of the diatonic pitch set from the circle of fifths would also be an argument to call the Dorian church mode naturally superior to the major scale, or to claim natural superiority—and the function of centre of gravity—for the supertonic (D in C major) because of its central position in the series:

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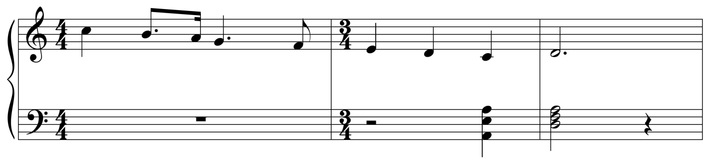
[65] Mark Devoto, in Don Michael Randel (ed.). *The New Harvard Dictionary of Music.* Belknap Press of Harvard University Press, 1986, p. 862, [my italics].
[66] Brian Hyer, *Tonality*, footnote p. 728, [my italics].
[67] Dmitri Tymoczko uses the term "centricity" to indicate "the phenomenon whereby a particular pitch is felt as being more stable or important than the others" (Dmitri Tymoczko, *A Geometry of Music; Harmony and Counterpoint in the Extended Common Practice*, Oxford University Press, 2011, p. 16).
[68] See: Bret Aarden, *Dynamic Melodic Expectancy*. Ph.D. dissertation. School of Music, Ohio State University, 2003; in: David Huron, *Sweet Anticipation, Music and the Psychology of Expectation*, MIT press, 2006, p. 148.
[69] See for instance: Jean-Philippe Rameau, *Treatise on Harmony*, Translated by Philip Gosset, Dover Publications, 1971, pp. 13-4.
[70] See for instance: Heinrich Schenker, *Harmony*, edited and annotated by Oswald Jonas, translated by Elisabeth Mann Borgese, The University of Chicago Press, 1954, p. 26.
[71] Richard Norton, *Tonality in Western Culture*, The Pennsylvania State University, 1984, p. 34.
[72] Richard Norton, *Tonality in Western Culture*, p. 34.

F C G D A E B

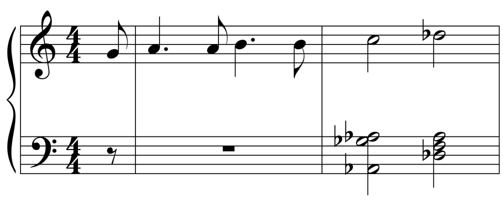
Robert Gauldin calls the tonic “the pitch class of greatest centrality, stability and finality in a tonal composition”[73] and tonality “the broad organization of pitches around the central tonic in a passage or piece of music”[74]. Not only is this a circular definition, it is also unclear why the tonic should be the pitch class of greatest centrality, stability and finality. Gauldin uses the carol *Joy to the World* as an example to illustrate his claim. Of course, it is well known what the tone of finality or conclusiveness is in this piece. It is the note the phrase ends on, but there is no reason why it could not have been another note. Imagine the first half phrase went up a second to D after the ‘final’ C, instead of stopping on that C. Especially with an alternative harmonisation like in Example 3.2, this would make D the tone of finality or conclusiveness, and the melody would all of a sudden become modal (Dorian). After hearing the melody this way a couple of times, it will even sound modal when heard without the harmonisation (which was not the case if the melody was known before and heard unharmonized without the alternative ending).



Example 3.2: First half phrase of *Joy to the World*
with different ending and alternative harmonisation.

Instead of this alternative ending, another ending on any of the pitch classes of the diatonic set of C major could be possible. This leads to the tautological conclusion that the tone of finality or conclusiveness can be any tone that concludes the melody, and that, although this tone is most commonly the tonic C in the set with all natural pitch classes, there is no reason why C necessarily has to be the pitch class of greatest centrality, stability and finality. In other words, being the pitch class with those characteristics is not what is usually understood as the tonic (D in Example 3.2 is usually not called a tonic, but the finalis in the Dorian mode). Gauldin too rightly remarks that “white-key melodies” (melodies using only the white keys of the piano keyboard) may “display a tonic other than C”[75], as is the case in the church modes. So, any note of the tonal diatonic set (or at least of the major diatonic set) may be perceived as a tonic.

Let us take the example of *Joy to the World* further. At the end of the first phrase of this carol, the C would not be conclusive if one would imagine it—and this is possible—as part of a dominant seventh chord on A flat, or if the phrase ended one note later on a conclusive D flat (a note that wasn’t even heard before), as in Example 3.3. A note has therefore only got great stability or finality if it ends the melody and is *perceived as such* (even if only in the perceiver’s imagination). The definition of tonality based on perception will be discussed below.



Example 3.3: Alternative ending to the first phrase of *Joy to the World*.

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[73] Robert Gauldin, *Harmonic Practice in Tonal Music*, W.W. Norton &C[ie], second edition, 2004, p. 33.
[74] Robert Gauldin, *Harmonic Practice in Tonal Music*, p. 34.
[75] Robert Gauldin, *Harmonic Practice in Tonal Music*, footnote p. 34.

According to the tonic-based definitions, the tonic is a central tone or a reference tone, but not any note can count as a tonic. Usually central tones as they occur in some atonal idioms, are not called 'tonic'. My own idiom, for instance, is structurally based on the constant use of central tones. My piano piece *Monodie* is, as was mentioned before,[76] entirely based on a Gauss distribution of clusters around pitch class A, but I call neither this central and gravitational pitch class, nor the other central tones occurring in my other pieces, a 'tonic' by any definition, and they would even less make my music tonal. At least, the central pitch classes in my compositions would not be 'tonics' the way the concept is generally understood.

What is implicitly assumed in the definitions of tonality is a very specific meaning of the concept of tonic. Tonic is generally defined as the "First degree *of the major or minor scale*."[77] Other definitions are formulated in terms of: "*In the major–minor tonal system*, the main note of a key (also called its key note), after which the key is named. […]. In music based on one of the church modes, the function of tonic is most closely approached by the FINAL of that mode."[78]

So, according to such definitions, a central tone is only called a tonic *when it is part of the major-minor system*. The ending note in the medieval modes, for instance, *closely approaches* the function of tonic (according to Grove Music), but is not considered to be a fully-fledged tonic. If the use of major and minor scales in the tonal system is not assumed in the definition of tonic, that would make the definitions of tonality and tonic circular and meaningless (because it would be applicable to any two related concepts), because then, the tonal system would be 'a system organized around a central tonic' and the tonic would be defined as 'the central note in the tonal system'. Why was the D flat in Example 3.3 perceived as "the pitch class of greatest centrality, stability and finality" by tonally trained listeners? Because it is a particular pitch class in a major or minor scale or in a church mode, in what I call a **tonal diatonic set (or set class)**[79] or **tonal 7-sets**[80] (the set classes with Forte numbers [7-32], [7-34] and [7-35])[81]. Therefore the addition of the major-minor system is a necessary element in the definition of tonality based on the occurrence of a tonic. It is the actual defining element within the definition.

3.2.3 Functionality and hierarchy

A third category of definitions of tonality contains those that claim that the tonic is not only a scale degree but also a harmonic function. The theory of functionality is a "theory of tonal harmony established by Hugo Riemann (1849-1919), who devised the term. The theory is that each chordal

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[76] See Section 1.1.1.
[77] "Tonic" in *The Oxford Dictionary of Music* (online), [last accessed: 28 November 2009], [my italics].
[78] "Tonic" in *Grove Music Online* [last accessed: 28 November 2009], [my italics].
[79] The term 'diatonic set (or diatonic pitch collection)' is here not only used in the restricted sense of "a group of seven pitch classes which forms the pattern of adjacent white keys on the piano" (Robert Gauldin, *Harmonic Practice in Tonal Music*, p. 33); its sense is also broader than the sense used in Grove's Dictionary, which states that "[a] seven-note scale is said to be diatonic when its octave span is filled by five tones and two semitones, with the semitones maximally separated, for example the major scale (T-T-S-T-T-T-S). The natural minor scale and the church modes are also diatonic" (Groves Music Online, lemma "Diatonic", [last accessed: 03 May 2013]). In the present context, the concept of **diatonic set** refers to any pitch class set of seven distinct pitch classes where each of the seven note roots appears exactly once (Note roots are represented by note names without accidental. C flat, C natural, and C sharp, for instance, belong to the same the same note root C. A pedal harp is tuned in diatonic sets, when no two strings are tuned on the same (enharmonic) pitch class—otherwise the strings of the harp do not from a pitch class set of cardinality 7). Another name for diatonic sets in this sense might be 'heptatonic sets'. **Tonal diatonic sets** are defined as the sets of the major scale and the minor scales. Gauldin's or Grove's diatonic set is what I call a major tonal diatonic set (including the natural minor scale and the church modes), the set with forte-number [7-35].
[80] The **tonic** may be defined as the second pitch class (1) in the prime form of the tonal diatonic sets (the prime forms are (013568T) for [7-35], (013468T) for [7-34], and (0134689) for [7-32]).
[81] see: Allen Forte, *The Structure of Atonal Music*, 1977, Yale University Press, pp. 179-181 for a complete list of numbered set classes.

identity within a tonality can be reduced to one of three harmonic functions—those of tonic, dominant and subdominant. Thus, for example, a supertonic chord has the function of a subdominant"[82]. Riemann "proposed that tonic, dominant and subdominant roots are of first importance in expressing a tonality."[83] So, functionality is not only a harmonic theory but the concept also applies to chords. Every scale degree in major and minor scales has a function but it is only in a harmonic context that the three functions become apparent.

A couple of remarks should be made in this respect. First of all, functionality suggests a hierarchical structure just like the predominance of the scale degree tonic does. Hierarchy-based definitions of tonality state that tonality is a system where pitch classes have different importance (the tonic usually being the most important). "When we speak of *tonality* we are referring to a […] phenomenon […] in which the relationships within a collection of tones orient that collection toward some single most important tone, the *tonic.* This phenomenon is hierarchic"[84]. The hierarchical structure of tonality lies at the basis of the idea that "Tonality is above all a language of pitch relationships"[85]. Hierarchy and pitch relationship however do not apply to tonal music alone. Modal music and many atonal idioms also possess a hierarchy in their pitch relations. In the adjusted Lydian and Dorian modes, for instance, the *finalis* and *tuba* are identical to the tonic and dominant of the major and natural minor scales and each have a specific function. Note also that dodecaphony, as an example of atonality, lies at the basis of idioms where all tones are related to each other, albeit not (necessarily) in a hierarchical way, and it is arguable whether this relationship is less essential than tonal relationships. Isn't pitch relationship an aspect of all pitched music? Pitch relationship is after all an important part of the structure of music, and structured sound is commonly accepted as a defining feature of music. Tonal functionality is therefore only one kind of functionality; one that implies the use of major and minor scales. "Fully 'emerged' tonality embraced a hierarchical system of relation between the tonic triad and the remaining six triads *of the diatonic scale*"[86].

Secondly, if functionality emerges from the relation between pitch classes or (tonal) chords it is context bound. This means there is no tonality when pitch classes or chords appear isolated. "A single pitch, interval, chord, or scalar passage cannot reign as tonic without a musical context that provides functional relations sufficient to determine its tonal meaning"[87]. Full tonality only emerges when the three harmonic functions have been heard.[88] In the light of this approach, the first 136 bars of Wagner's *Rheingold* cannot be called tonal since they are based on only one (E-flat major) triad. This, of course, is not how the *Vorspiel* to *Das Rheingold* is commonly interpreted; not even by people who support functionality definitions of tonality. Most tonally acculturated listeners perceive the first 136 bars of the *Vorspiel* to *Das Rheingold* as being tonal, even though they contain only one chord and therefore only one tonal function. As will be discussed below, this is because the 136 bars are built exclusively with the pitch classes of the tonal diatonic set of E flat major. In Stravinsky's *Sacre du Printemps* too, it is often hard or impossible to hear all tonal functions because of its "harmonic stasis"[89], but stasis (e.g. in the *Augurs* chord) itself is a feature of the tonic function, the function of rest and finality. Whenever the *Sacre* is clearly diatonic, at least a tonic can be perceived. I therefore consider the *Sacre* as tonal when it is diatonic, even though it is not completely functional. Sub-dominant and dominant functions are often missing and therefore functional harmonic analysis is ineffectual or pointless most of the time.[90]

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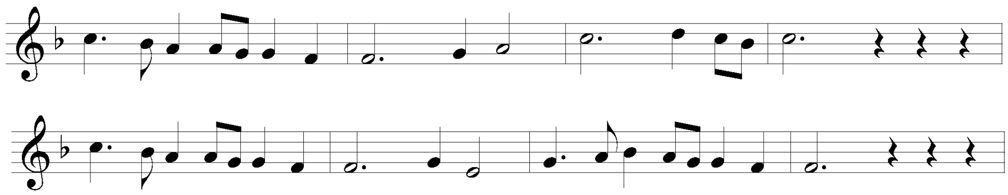
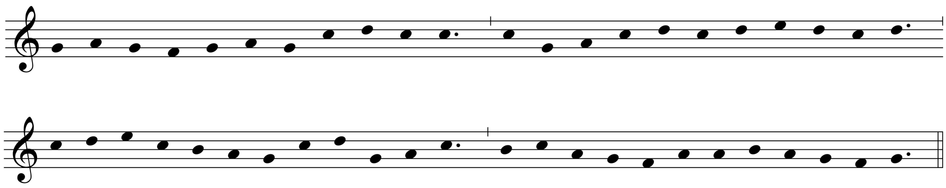
[82] Alison Latham (ed.), *The Oxford Companion to Music*, 2002, Oxford University Press, p. 496.
[83] Alison Latham (ed.), *The Oxford Companion to Music*, pp. 1280-1.
[84] Lee Humphries, *Atonality, Information, and the Politics of Perception*, 2000, p. 1, in http://www.thinkingapplied.com/tonality_folder/tonality.pdf [last accessed: 14 July 2010].
[85] Joel Lester, *Analytic Approaches to Twentieth-Century Music*, W.W.Norton and company, 1989, p. 4.
[86] Richard Norton, *Tonality in Western Culture*, The Pennsylvania State University, 1984, p. 24 [my italics].
[87] Helen Brown, *The Interplay of Set Content and Temporal Context in a Functional Theory of Tonality Perception*, in *Music Perception*, Vol. 5, N°3, 1988, pp. 245.
[88] Note that when the chords on the first (tonic), fourth (subdominant) and fifth degree (dominant) have sounded, all the pitch classes of the tonal 7-sets have sounded.
[89] Jonathan Cross (ed.), *The Cambridge Companion to Stravinsky*, Cambridge University Press, 2003, p. 88.
[90] For an extended discussion about the tonality or atonality of the *Sacre* between Allen Forte and Richard Taruskin, see: Richard Taruskin, *Letter to the Editor from Richard Taruskin*, in Music Analysis, Vol. 5, N°. 2/3, 1986, pp. 313-320.

Finally, if it is only in a harmonic context that the three functions become apparent, what then to say about Bach's suits for cello solo or other monophonic music? There is no explicit harmony in monophonic music. The harmonic context is only present in an *implicit* way. It is hidden, and it can only be 'imagined' or 'supposed' by the listener? But what then with 'imagined' or 'supposed' harmony in modal music? For a tonally acculturated listener it is almost impossible not to imagine functional harmony in pieces like the one in the example below (Example 3.4):



Example 3.4: Canso *Porque Trobar*.

It is hard not to 'imagine' this piece in F major and not to 'hear', for instance, the dominant function (even the dominant seventh chord) in the second last bar of Example 3.4. So, the melody in this example is perceived as functional by tonally acculturated listeners, although it is modal (it is the transcription of the medieval Canso *Porque Trobar*)[91]. It is indeed possible for a tonally trained (and biased) listener to hear the Gregorian hymn *Veni Creator Spiritus* in Example 3.5 ending on a dominant in C major instead of the hypomixolydian mode (mode VIII).



Example 3.5: Gregorian hymn *Veni Creator Spiritus*.

This suggests that functionality is not intrinsic to a piece of music, but is a feature of perception by tonally cultivated listeners; a possible feature not only of tonal music, as contrasted with modal music, but of *all diatonic music*. As Wallace Berry states:

> The concept of the hierarchic ordering of pitch content has in one manifestation or another served as a basis for musical structure since the earliest stages in Western tradition. […] The practice or hierarchic systems of tonal order [engenders] the concept of tonality […]. Tonality may thus be broadly conceived as *a formal system in which pitch content is perceived as functionally related to a specific pitch-class or pitch-class-complex of resolution*, often pre-established and preconditioned, as a basis for structure at some understood level of perception. The forgoing definition of tonality is applicable not just to the "tonal period" in which the most familiar conventions of tonal function are practiced (roughly the eighteenth and the nineteenth centuries), but through earlier modality and more recent freer tonal applications as well.[92]

[91] This example is an excerpt from a larger version appearing in Richard Taruskin, *The Oxford History of Western Music, Volume 1: Music from the Earliest Notations to the Sixteenth Century,* Oxford University Press, 2010, p. 129.
[92] Wallace Berry, *Structural Functions in Music*, Dover Publications, 1976, p. 27.

According to Daniel Harrison the tonal functions "Dominant, Tonic, and Subdominant are harmonic labels denoting perceptual impressions"[93]. Harrison adds that the idea of harmonic function is "elusive and equivocal and has been from its origins. The inventor of the term, Hugo Riemann, was never quite clear himself about what a harmonic function is, and his confusion inspired many subsequent authors to attempt clarifications and refinements that unfortunately, in too many cases, trapped the idea further in a sticky web of ambiguity"[94].

3.2.4 Perception

The end of the previous section leads us to the category of perception-based definitions of tonality. The definitions in this category are formulated in terms of: "When we speak of *tonality* we are referring to a *perceptual phenomenon*"[95]. Such perception-based definitions of tonality refer to the already discussed aspects of relationship, hierarchy and functionality, which all relate to the presence of the diatonic scales; yet, they differ from the other definitions in the sense that they see perception as a defining element of tonality.

Perception is intentional; it is always the perception *of* something, something different from the act of perception (the Husserlian *noesis*). If tonality is perception then it is the (subjective) perception of something else; some (objective) feature of the music perceived. Tonality cannot be both a feature of music and the perception of that feature at the same time. Therefore, in order to avoid confusion and misunderstanding, a choice has to be made as to what will be called tonality. In the present context, tonality is considered the object, cause or source of perception, so it is not the perception itself, and therefore—although tonality may be perceived is a specific way—perception is not part of the definition of tonality in the present context.

3.2.5 Atonality

Tonality is, as was discussed above, sometimes defined in terms that make the distinction between music from the common-practice era (ca. 1600-1900) and music from Middle Ages and the Renaissance possible. This distinction is convenient when only these two eras are discussed. It is insufficient, however, when music commonly referred to as 'atonal' enters the picture. As was mentioned in the introduction, the purpose of the present section is to define tonality in such a manner that the definition is applicable in an atonal context as well as in a context including Early music. An overview of some definitions of atonality is therefore necessary before attempting to define tonality in a way that will serve the intended purpose.

Although Arnold Schoenberg's claim that "The word 'atonal' could only signify something entirely inconsistent with the nature of tone"[96] is etymologically correct, it does not supply a useful definition within the context of the present dissertation, because it makes the meaning of the concept 'atonal' so broad that it includes virtually all music based on the occurrence of tone.

The most suitable definitions of atonality in the present context are formulated in terms of: "Atonality […] literally means 'without tonality'. What it should mean is 'without a diatonic concept of tonality'"[97], or: "'Atonal' music refers to that music which is not clearly organized by traditional

[93] Daniel Harrison, *Harmonic Function in Chromatic Music, A Renewed Dualist Theory and an Account of its Precedents*, The University of Chicago Press, 1994, p. 36.
[94] Daniel Harrison, *Harmonic Function in Chromatic Music*, p. 37.
[95] Lee Humphries, *Atonality, Information, and the Politics of Perception*, 2000, p. 1, in http://www.thinkingapplied.com/tonality_folder/tonality.pdf [last accessed: 14 July 2010] [my italics].
[96] Arnold Schönberg, *Harmonielehre*, Universal Edition, 1911. Translation by Roy E. Carter: *Theory of Harmony*, Faber & Faber, 1983, p. 432.
[97] Ludmila Ulehla, *Contemporary Harmony*, Advance Music, 1994, p. 484.

systems, such as the modal system, or the major and minor key systems"[98]. Definitions of this kind lead to concepts of tonality and atonality as opposites. Again, they refer (explicitly this time) to the tonal diatonic sets. Atonality is sometimes defined on the basis of a blatant misconception of what atonal music is about. In a radio interview in 1930, Alban Berg said:

> [T]he word *atonal*—I must add, unfortunately—came to stand collectively for music of which it was assumed not only that it had no harmonic center (to use tonality in Rameau's sense), but was also devoid of all other musical attributes such as melos, rhythm, form in part and whole; so that today the designation as good as signifies a music that is no music, and is used to imply the exact opposite of what has heretofore been considered music.[99]

But, he adds, "[e]ven if certain harmonic possibilities are lost through abandonment of major and minor, all the other qualities we demand of a true and genuine music still remain"[100]. Berg claims: "the word atonal must be a misnomer for this tendency in music"[101], and: "the word 'atonal', which is equivalent to anti-musical, ugly, uninspired, ill-sounding and destructive."[102] Indeed, even today, the term 'atonal' is still all too often defined in terms of "unattractive", "incomprehensible", "unstructured", "meaningless" or simply "ugly" even by (future) professional musicians.[103] It should be clear that this is by no means the sense in which "atonality" is to be understood in the present context. The term is used without a connotation based on aesthetic judgment. It is indeed evidently not my intention to compose "anti-musical, ugly, uninspired, ill-sounding or destructive" music.

3.2.6 Common-practice

Definitions of he concept of tonality are sometimes restricted to the use of tonality in common-practice. Common-practice-based definitions do not say what tonality *is*, but what the general features of music belonging to that practice are. Joseph Straus lists six characteristics that define "traditional common-practice tonality, the musical language of the Western classical music from roughly the time of Bach to roughly the time of Brahms"[104]:

> 1. *Key*. A particular note is defined as the tonic (as in "the key of C [sharp]" or "the key of A") with the remaining notes defined in relation to it.
> 2. *Key relations.* Pieces modulate through a succession of keys, with the keynotes often related by perfect fifth, or by major or minor thirds. Pieces end in the key in which they begin.
> 3. *Diatonic scales*. The principle scales are the major and the minor scales.
> 4. *Triads.* The basic harmonic structure is a major or minor triad. Seventh chords play a secondary role.
> 5. *Functional harmony.* Harmonies generally have a function of a tonic (arrival point), dominant (leading to tonic), or predominant (leading to dominant).

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[98] Reginald Smith Brindle, *Serial Composition*, Oxford University press, 1966, p. 11.
[99] Quoted in Bryan R. Simms, *Composers on Modern Musical Culture: An Anthology of Readings on Twentieth-Century Music*, Wadsworth Publishing Co Inc, 1999, p. 63. See also: Alban Berg, *Was ist Atonal?,* 23: Eine Wiener Zeitschrift 26/27, 1936, pp. 1-11, and Alban Berg, *Ecrits,* Dominique Jameux (ed.), Henri Pousseur, Gisela Tillier & Dennis Collins (translators), Christian Bourgois Editeur (Paris), 1985, pp. 52-3 [original italics].
[100] Quoted in Bryan R. Simms, *Composers on Modern Musical Culture,* p. 64.
[101] Bryan R. Simms, *Composers on Modern Musical Culture*, p. 67.
[102] Quoted in Bryan R. Simms, *Composers on Modern Musical Culture*, p. 68.
[103] Year after year, several of my music analysis students at the LUCA (campus Lemmens, Leuven University) describe atonality in these terms in their written assignment on tonality and atonality.
[104] Joseph N. Strauss, *Introduction to Post-Tonal Theory*, Pearson Prentice Hall, 3rd edition, 2005, p. 130.

> 6. *Voice leading*. The voice leading follows certain traditional norms, including the avoidance of parallel perfect consonances and the resolution of intervals defined as dissonant to those defined as consonant.[105]

More recently, Dmitri Tymoczko argued that "five features are present in a wide range of genres, Western and non-Western, past and present, and that they jointly contribute to a sense of tonality"[106]. These features are:

> 1. *Conjunct melodic motion*. Melodies tent to move by short distances from note to note.
> 2. *Acoustic consonance*. Consonant harmonies are preferred to dissonant harmonies, and tend to be used at points of musical stability.
> 3. *Harmonic consistency*. The harmonies in a passage of music, whatever they may be, tend to be structurally similar to one another.
> 4. *Limited macroharmony*. I use the term "macroharmony" to refer to the total collection of notes heard over moderate spans of musical time. Tonal music tends to use relatively small macroharmonies, often involving five to eight notes.
> 5. *Centricity*. Over moderate spans of musical time, one note is heard as being more prominent than the others, appearing more frequently and serving as a goal of musical motion.[107]

Of the listed features, several have already been addressed: the features of "key" and "centricity" relate to the idea of tonic discussed above, and the feature of "functional harmony" was addressed in the discussion of functionality and hierarchy. They belong to more general, not necessarily common-practice-related definitions of tonality.

"Key relations", "triads", and "acoustic consonance" are indeed features of music belonging to music of the common-practice, as Straus an Tymoczko rightly claim, but they do not *define* tonality; that is, music does not necessarily cease to be tonal if one or all of those features are absent. Pieces with "distant" modulations or piece that end on a different key than the one they started in, for instance, are not atonal (or non-tonal) for that reason. As will be discussed later, the concept of tonality and consonance are independent and—although it was the case in common-practice—music does not have to be highly consonant in order to be tonal.

A similar remark can be made about the feature of "voice leading" or "conjunct melodic motion". Although it is true that common-practice music is characterized by "a preference for *conjunct melodic motion*"[108], and although it may be true that it is harder for the human 'ear' to perceive melody when notes are spread out over larger intervals, this does not entail that deviations from the 'traditional norms' of voice leading in common-practice music necessarily result in music that cannot be called tonal any longer. Irregular resolutions of dissonances in a tonal context too would be just that: 'irregular *within a tonal context*'; they would not necessarily make the context atonal. As to the "avoidance of parallel perfect consonances" mentioned by Straus: again it is true that in common-practice those parallel consonances are avoided (they are even forbidden in academic harmony), the presence of parallel perfect fifths and octaves, may be a sign of poor academic tonal harmonisation, it doesn't make chord succession less tonal nevertheless, as the following excerpt from an exercise by a beginner student illustrates (see Example 3.6)[109].

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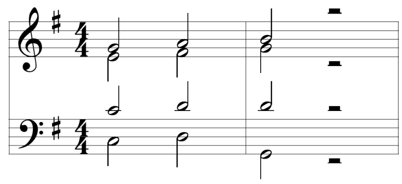
105 Joseph N. Strauss, *Introduction to Post-Tonal Theory*, p. 130, [my italics for reasons of conformity with Tymoczko's list of features].
106 Dmitri Tymoczko, *A Geometry of Music; Harmony and Counterpoint in the Extended Common Practice*, Oxford University Press, 2011, p. 4.
107 Dmitri Tymoczko, *A Geometry of Music*, p. 4.
108 Dmitri Tymoczko, *A Geometry of Music*, p. 5 [original italics].
109 This is an excerpt from an actual first harmony exercise of one of my pupils.



Example 3.6: Harmonisation with parallel fifths and octaves.

Absent in the lists of Straus and Tymoczko are the ideas of characteristic rhythm, stress and meter in common-practice music. These are addressed by (a.o.) Fred Lerdahl and Ray Jackendoff in *A Generative Theory of Tonal Music*[110], or by Robert Gauldin, who claims that the sense of finality of the tonic is reinforced by "(1) the frequent occurrence of [the tonic] through repetition[111], (2) the frequent placement of [the tonic] on the downbeat, stressing it metrically, (3) the tonal reinforcement of [the tonic] by preceding it with [the dominant], producing a perfect 4^{th} up or a perfect 5^{th} down (in both cases the root of the interval is [the tonic], and (4) the strong tendency of the [leading tone] to resolve upward by half step to the [tonic]"[112]. Again, these features—at least features 2 through 4—may indeed determine which note will be perceived as a tonic, but it does not determine whether the piece is tonal or not, as long as the piece is mainly based on a tonal diatonic set; they are, in other words, no defining features of tonality. Deviations from these traditional norms do not necessarily result in atonality. Only the last feature of music belonging to the common-practice occuring in the lists above: "diatonic scales" (Straus) and "limited macroharmony" (Tymoczko) are defining features of tonal music.

3.3 Diatonicity and degree of tonality

3.3.1 Diatonicity as a necessary condition for tonality

In all the definitions of tonality discussed above the idea of tonal diatonicism—the feature of music with a macroharmony based on a single tonal diatonic scale—played an important role: the concepts of tonic, of tonal functionality and hierarchy, and of perception of tonality, were all based on the presence of the tonal diatonic set (in the sense defined above). The features of common-practice tonality were also based on diatonicism. Diatonicism-based definitions of tonality explicitly state that tonal music is music that is determined by the presence of major and minor scales, as was explicitly the case in several of the definitions stated above (and implicitly also in Smith Brindle's definition of atonality as the antonym of tonality).[113] Diatonicism-based definitions of tonality are sometimes restricted to "[t]he major/minor system of pitch organization that has dominated Western music since the 16^{th} century ([and that] contrasts with modality and atonality)"[114], but sometimes also include "music based on, among other theoretical structures, the eight ecclesiastical modes of medieval and Renaissance liturgical music"[115]. In the present context, the concept of tonality is used in a combination of both senses, including modality but contrasting with atonality.

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[110] See Fred Lerdahl & Ray Jackendoff, *A Generative Theory of Tonal Music*, MIT Press, 1983. The "Well-Formedness Rules" developed in this book correspond to the traditional norms of tonal perception of common-practice music.

[111] As was discussed above, this claim was refuted by Bret Aarden.

[112] Robert Gauldin, *Harmonic Practice in Tonal Music*, W.W. Norton &C^{ie}, second edition, 2004, pp. 34-5.

[113] This also applies to, for instance, Gottfried Weber's theories. See: Brian Hyer, *Tonality*, in Thomas Christensen (ed.)*, The Cambridge History of Western Music Theory*, Cambridge University Press, 2002, p. 727, referring to Gottfried Weber, *Versuch einer geordneten Theorie der Tonsetzkunst* (1817-21), 3^{rd} edition, B. Schott, 1830-32, Translated by J. Warner as The Theory of Musical Composition, R. Cocks, 1851.

[114] David Huron, *Sweet anticipation. Music and the psychology of expectation*, MIT press, 2006, pp. 421-2.

[115] Brian Hyer, *Tonality*, in Thomas Christensen (ed.)*, The Cambridge History of Western Music Theory*, p. 727.

The necessary presence of a diatonic scale in the definitions of tonality discussed above make diatonicism a necessary condition for tonality. Tonal music is diatonic music. This condition is too strict, however. Most music belonging to the common-practice—or other music that is regarded as tonal according to the customary definitions—is not *strictly* diatonic. Although, as Tymoczko rightly claims, most tonal music (certainly the music belonging to common-practice) has a limited macroharmony, since it is mostly based on a single tonal diatonic set "over a moderate span", tonal music without so-called chromatic notes (the five pitch classes that do not belong to the tonal diatonic set used) within that moderate span, or without chromatic changes that end in modulation—moving the piece from one tonal diatonic set to another (often closely related)[116] one—is very rare. But even if music is not strictly diatonic, it can still be called 'tonal' if its macroharmony is closely enough related to a single tonal diatonic set, as is generally the case in common-practice music. The level or degree of relation or similarity of a pitch class set to a tonal diatonic set is what I call the **degree of diatonicity** of that set or of the music which has the set in its macroharmony. The higher the similarity of a pitch class set to a tonal diatonic set, the higher its degree of diatonicity. I will claim that tonality is determined by (degree of) diatonicity. Not diatonicism but (a high enough degree of) diatonicity is the necessary condition for tonality stated in the definitions discussed above. Indeed, without (a high enough degree of) diatonicity—that is: if the macroharmony of a piece of music is too different from a single tonal diatonic set—there would be no tonic (or no *finalis* in the case of modal music), no tonal functionality or hierarchy, and no perception of tonality, even if the other traditional norms of common-practice are respected.

3.3.2 Diatonicity as a sufficient condition for tonality

In order for tonality to be fully determined by diatonicity, it is not enough that diatonicity is a necessary condition for tonality; it also has to be a sufficient condition. In other words, a high resemblance of a piece's macroharmony with a single tonal diatonic set has to be enough for the piece to be perceived as tonal. Diatonicity as a necessary condition means that music can only be tonal if its macroharmony bears enough resemblance to a tonal diatonic set; as a sufficient condition it means that as soon as this resemblance is high enough, the piece will be called tonal and perceived as such by tonally acculturated individuals. The fact that the listener has to be acquainted with the tonal system and with functionality is important in this respect. As David J. Hargreaves claims:

> Winner (1982) speculates that there is probably nothing special about Western scales, in the sense that children growing up in other societies are likely to acquire their own scales in a similar manner. The acquisition of tonality is thus comparable with language acquisition, in that the general capacity to master language is a maturational one that is independent of exposure to and training in the particular language acquired.[117]

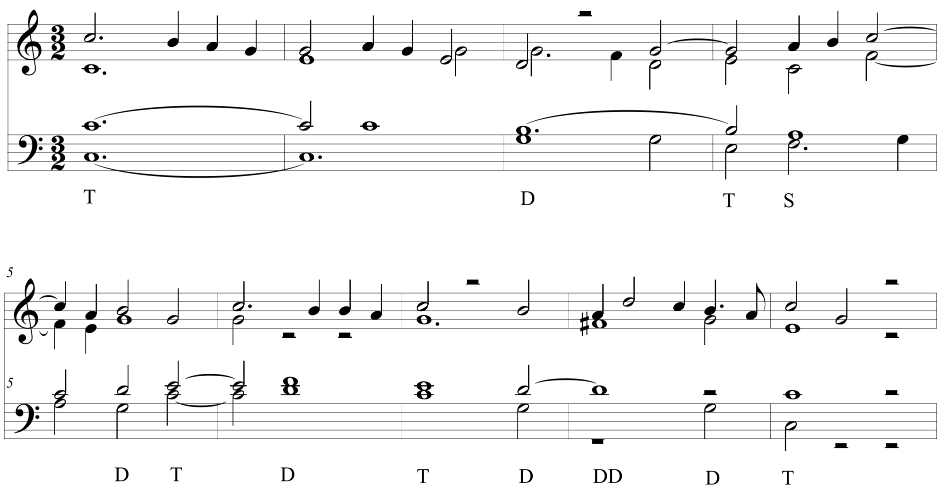
This suggests that without acculturation, listeners will not perceive functionality or other features belonging to the tonal system, just like listeners who are not acquainted with Indian traditional music are unable to identify ragas. Conversely, as was observed in the section on functionality and hierarchy, it is arguably impossible for a tonally acculturated listener not to hear functions in any diatonic music. In Daniel Harrison's words:

> The dualism of major and minor systems takes diatonic scale structure as [a] shared characteristic. Diatonicism[118], the bedrock of Western theories of scale for both modal and major-minor tonalities, is largely responsible for our ability to perceive harmonic function. Because of the special intervallic properties of the diatonic scale, which allow us to deduce the entire structure from just a few notes,

[116] See Straus's "key relations" (feature 2) discussed above.
[117] David J. Hargreaves, *The Developmental Psychology of Music*, Cambridge University press, 1986, pp. 91-2, referring to Ellen Winner, *Invented Worlds: The Psychology of the Arts*, Harvard University Press, 1982.
[118] Diatonicism is here to be understood as diatonicity, as becomes clear in the rest of the quote.

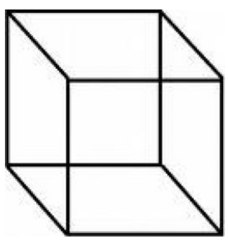
we are able easily to experience the tonal centricity on which harmonic function depends.[119]

Therefore, functionality can be claimed to be a feature of all highly diatonic music (music with a high enough degree of diatonicity), making diatonicity a sufficient condition for the perception of tonality by tonally acculturated listeners, even if the music is not conceived functionally. Indeed, it is arguably impossible for a tonally acculturated listener not to perceive functions in the following example taken from the beginning of the Kyrie from Guillaume Du Fay's *Missa Se la face ay pale*, even if it is considered historically wrong to analyse this piece the way it is done in Example 3.7, because it predates functional harmonic thinking.



Example 3.7: Functional analysis of the beginning of the Kyrie from Guillaume Du Fay's *Missa Se la face ay pale.*

This example was chosen completely ad random[120], but the claim that we cannot help but hear tonal functions in modal music or other non-functional tonal music probably stands for all existing modal pieces as well as other non-functional highly diatonic pieces, as a result of perceptual categorization in the process of feature analysis happening in the auditory cortex of tonally acculturated listeners.[121] In visual perception, the cerebral process of categorization causes the fact that we cannot help but "see" the two-dimensional picture below (Example 1.35) as (a representation of) a three-dimensional cube.[122]



Example 3.8: Two-dimensional figure with twelve lines interpreted as a three-dimensional cube.

[119] Daniel Harrison, *Harmonic Function in Chromatic Music, A Renewed Dualist Theory and an Account of its Precedents*, The University of Chicago Press, 1994, pp. 24-5.

[120] Other examples (the Canso *Porque Trobar* and the Gregorian hymn *Veni Creator Spiritus*) were discussed above in the section on perception of functionality and hierarchy.

[121] See: Douglas A. Bernstein, Edward J. Roy, Thomas K. Srull, Christopher D. Wickens, *Psychology*, Houghton Mifflin Company, 2nd edition, 1991, p. 196.

[122] See also: Marcel Danesi, *Messages, Signs, and Meanings: A Basic Textbook in Semiotics and Communication Theory*, Canadian Scholar's Press Inc., 3rd edition, 2004, p. 77.

Once acculturated listeners have discerned functions it is hard or impossible for them not to hear the functions afterwards. The following example illustrates this phenomenon of perceptual categorization in the process of visual feature analysis.[123] The picture below (Example 3.9) shows black spots on a white surface. It may be hard to detect what the picture represents at first, but once the viewer has spotted the sniffing dog (a Dalmatian) it becomes impossible for him or her not to see it. “Basically, the brain must analyze the incoming pattern of light and compare the pattern to the information stored in memory. If it finds a match, recognition takes place and the stimulus is placed into a perceptual category. Once this recognition occurs, your perception of a stimulus may never be the same.”[124]

Example 3.9: Perceptual categorization in visual feature analysis
(Source: image by R. C. James in David Marr, *Vision*,
W. H. Freeman and Company, 1982 re-edition MIT Press, 2010, p. 101).

The same phenomenon occurs in language. When we hear a language we do not know, we perceive only its sound, without understanding what is said. In some cases we cannot even identify which language is spoken, or if what is said is a linguistic utterance. Once we get familiar with the language, however, we can no longer hear it without ‘hearing’ verbs or nouns, and without attaching meaning to the words and sentences uttered. In Lewis Carroll’s nonsensical poem *Jabberwocky* in *Through the Looking-Glass*[125], in James Joyce’s *Finnegans Wake* with its abundant neologisms, or in the primeval sounds of Kurt Schwitters’s *Ursonate* (1922-1932), sounds that accidentally correspond to words in a familiar language will be heard as such, even if they were not meant to be the expressions of existing words (for instance if they form words in a language that was not known by Kurt Schwitters). If “Fümms”, the first sound in the *Ursonate*,[126] for instance, were a meaningful word in some existing natural language, the speakers of that language would probably automatically and involuntarily understand that word with its linguistic meaning (its dictionary definition). This is certainly the case—and the possible cause of protest—if the accidentally generated meaningful words are taboo or considered inappropriate by speakers of that language. James Joyce’s *Finnegans Wake* forms a special case of this idea, in that the word “quark” has acquired a very specific meaning when in 1963 the Physicist Murray Gell-Mann used the actual word from the book as a name for the elementary particle he predicted.[127] There is probably no nuclear physicist today who can read or hear the verse *Three quarks for Muster Mark*[128] without involuntarily making a semantic referential link of some kind with

[123] See: Douglas A. Bernstein, Edward J. Roy, Thomas K. Srull, Christopher D. Wickens, *Psychology*, Houghton Mifflin Company, 2nd edition, 1991, p. 196.
[124] Douglas A. Bernstein, e.a., *Psychology*, p. 196.
[125] Lewis Carroll, *Alice Through the Looking-Glass, & What Alice Found There*, in The Complete Lewis Carroll, Wordsworth Editions, 1996, pp. 137-8.
[126] For a recording of Kurt Schwitters’ *Ursonate*, see: http://www.costis.org/x/schwitters/ursonate.htm [last accessed: 12 May 2013].
[127] See: Eugene Hecht, *Physics*, Brooks/Cole Publishing Cie. 1994, p. 1148.
[128] James Joyce, *Finnegans Wake*, Penguin Books, 1992, p. 383.

the elementary particle, although this specific association could never have been intended by James Joyce.

Mutatis mutandis, what was said about functionality can also be applied to the other aspects of tonality discussed above. For tonally acculturated listeners, highly diatonic music involuntarily entails the perception of a pitch class of greatest centrality, stability and finality. In the majority of cases this will be the ‘tonic’ (pc 1 in the prime form of the three tonal diatonic sets), but, as Robert Gauldin claims, any of the seven pitch classes of the tonal diatonic set may serve as well, making the piece modal. Other pitch classes will be perceived as having other functions within the piece. Again, this is a result of acculturation. It is not a natural feature of the leading tone to create expectations of closure, for instance, but “[a]s soon as the leading tone appears, *the acculturated ear* will anticipate the implied arrival”[129].

The perception of tonality is strongest in strictly diatonic music, using *all* the pitch classes of the tonal diatonic set and no others, regardless of the traditional norms of common-practice tonality. Indeed, regardless of voice leading, chord construction, acoustic consonance, harmonic consistency, frequency of occurrence, placement, or metrical stress of the tonic or other scale degrees, diatonicism (strict diatonicity) involuntarily leads to the perception of tonality for tonally acculturated listeners.

Although it is true that common-practice “melodies tend to move by short distances from note to note”[130], there are no restrictions to how a strictly diatonic tonal melody *could* move. Imagine any strictly diatonic tonal melody up to a certain note. Whatever came before, there is no limit to the possible notes that can follow, as long as they belong to the tonal diatonic set that characterised the macroharmony of the tonal melody so far. Although, as a result of tradition and acculturation, it is often possible to ‘guess’ the following note or notes of the melody, any pitch class of the tonal diatonic set in question would be possible. After a leading note, listeners acquainted with the tonal tradition expect a tonic (the regular resolution of the ‘dissonant’ leading note) to follow, but if another note of the piece’s diatonic scale would follow, the melody would still be perceived as tonal (including modality) by tonally acculturated listeners. In common-practice, “some successions of scale degrees are more likely than others”, but the probability for any pitch class set in diatonic continuation is never zero, as David Huron shows.[131] Some successions may be rare or even prohibited in certain styles or genres in common-practice, but their occurrence would only change the genre or style of the tonal piece, not the fact that it is tonal.

Even if the intervals were spread out over large intervals, as is the case in the third bar of the following excerpt from Gustav Mahler’s *9th Symphony* (finished in 1909) (see the third bar in Example 3.10)[132], the melody remains tonal and is perceived as such. Therefore, not only can any pitch class belonging to a tonal diatonic set follow any other pitch class, but any pitch can follow any other. Melodic consistency and common-practice recognition may be destroyed this way, but not tonality.

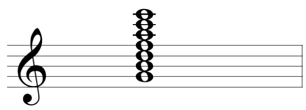
Example 3.10: Gustav Mahler, *Symphony N°9*, 4th movement bar 70-73.

[129] Susan McClary, *Towards a History of Harmonic Tonality*, in *Towards Tonality*, Collected writings of the Orpheus Institute, Leuven University Press, 2007, pp. 94-5, [my italics].
[130] Dmitri Tymoczko, *A Geometry of Music*, p. 4.
[131] The probability is only 0 in some cases where the “antecedent state” doesn’t belong to the diatonic scale (see Table 9.2 in David Huron, *Sweet anticipation, music and the psychology of expectation*, MIT press, 2006, p. 158. This table is based on “an analysis of more than a quarter of a million tone pairs” (p. 158) taken from “a sample of several thousand Germanic folk songs in major key” (p. 158), but since they belong to common-practice the results can be applied to all tonal music: the probability may be lower or higher for the whole of tonality, but it can never become zero).
[132] Only the first three bars of this melody are strictly diatonic. The A flat in the last bar disrupts the pattern.

What applies to voice leading can also be said about chord succession. The idea that "'Any chord can follow any other chord' [also in tonal music] promises a musical experience that we can associate more readily with Cage, for example, than with Reger. That we can still hear Reger's music as familiarly tonal in some sense makes this aphorism a much more complicated statement than at first it seems, for it means not only that any chord can follow another but also that a listener can make tonal sense of such a progression – indeed not merely *can* but *must*."[133]

To prove my claim that diatonicism—and by extension diatonicity—is a sufficient condition for tonality independent of voice leading, I developed an experiment with a 'random tonal music generator'. The experiment uses a Max/MSP patch (see Appendix 4) that generates a continuous 'melodic line' of random pitches belonging to a single tonal diatonic set (C major) within a range of two octaves[134] with random rhythm (without metre). The resulting melodic lines are clearly perceived as tonal (or modal)—not only by me but also by peers who underwent the test—as soon as most of the pitch classes of the set are heard. One pitch class (B) is generally perceived as leading note and harmonic functions are discerned.[135] This proves that the traditional norms for voice leading, frequency of occurrence of pitch classes, metric stress, phrasing, and the other common-practice features discussed above, are no necessary conditions for tonality.

An additional test with polyphonic music was performed during an experiment session on hyperpolyphony with professional composers and performers[136]. For this experiment I had composed three *Polytonal variations* (See scores in Appendix 5. I will return to these pieces later, in the section on polytonality (see Section 3.6)). During the discussion on the link between diatonicity and perception of tonality that followed the performance, the performers were asked to play the first *Variation* again, but without key signature. This resulted in strictly diatonic music using all the pitch classes of the set of C major and only those. Although the three parts in the piece were composed totally independently, avoiding all possible traditional metric features and harmonic functions, the piece was perceived as tonal (or modal) by all of the participants. Many of the harmonies in the C major version of the first *Polytonal variation* were dissonant, and without a harmonic structure based on major or minor triads of common-practice music mentioned by Joseph Straus. Consonance and triadic harmony, therefore, has no decisive influence on tonality. As will be discussed later, (tonal) consonance is a feature independent of tonality. In fact, any combination of pitch classes belonging to a single tonal diatonic set may be used to form a chord within a tonal context. All pitch classes may even be used together, resulting in a dominant thirteenth chord. Example 3.11 shows a dominant thirteenth chord on G, containing all the pitch classes of the set of C major.



Example 3.11: Dominant 13th chord on G.

[133] Daniel Harrison, *Harmonic function in chromatic music*, University of Chicago Press, 1994, p. 7.
[134] A possible extension of the range has no influence on the perception of tonality; it may only make the perception of melodic lines more difficult in some cases.
[135] Sometimes other notes are perceived as 'principle or referential' tone, and the melodic line is then perceived as modal.
[136] This session (held in the Conservatory of Ghent 8 March 2011) was part of the doctoral research of Hans Roels on Hyperpolyphony.

3.3.3 Degree of tonality

In the previous discussion, it was shown that a high degree of diatonicity is a necessary and sufficient reason for tonality.[137] If the macroharmony of a piece (or a section of the piece) has a macroharmony that is highly similar to one of the tonal diatonic sets (instances of set classes [7-32], [7-34] or [7-35]), it will automatically be perceived as tonal—with all the characteristics that are related to tonality—by tonally acculturated listeners, even if it lacks the customary features of common-practice music. Conversely, music will not be perceived as tonal unless its macroharmony has a high similarity with one of the tonal diatonic sets, even if the other traditional norms of common-practice are respected. The higher the degree of diatonicity of a piece's macroharmony, the stronger the perception of tonality will be. The more the macroharmony deviates from the tonal diatonic set, the lower its degree of diatonicity and the harder it becomes to perceive tonality. A very low degree of diatonicity results in what is called "atonal music", in Reginald Smith Brindle's sense that lies at the basis of the present dissertation: "'Atonal' music refers to that music which is not clearly organized by traditional systems, such as the modal system, or the major and minor key systems"[138]. Note that 'perfect tonality' is possible when the macroharmony of a piece coincides completely with a tonal diatonic set (that is, in case of diatonicism); 'perfect atonality' on the other hand is impossible since the presence of one tone (one pitched sound) makes the macroharmony of a piece already minimally similar to a tonal diatonic set (a single tone is a subset of all types of tonal diatonic sets). Music is never completely devoid of diatonicity. This recalls Arnold Schoenberg's claim that "[t]he word 'atonal' could only signify something entirely inconsistent with the nature of tone"[139]. But whereas Schoenberg rejects the term 'atonal' altogether when it is "intended in an exlusive rather than an inclusive sense"[140], in the sense used in the present dissertation, only complete, strict, or perfect atonality is impossible as soon as a single tone is involved.

One may ask whether it is possible to write atonal music (music with a very low degree of diatonicity) using exclusively all pitch classes of a single tonal diatonic set, and conversely, whether it is possible to write tonal music that has not got—at least locally—a high degree of diatonicity? If both questions are answered in the negative, high degree of diatonicity by contrast again proves to be both a necessary and sufficient condition for tonality. All my attempts to find one counterexample have failed up to date.

Until a counterexample has been found, it is safe to claim that the perception of tonality and atonality is determined by the degree of diatonicity of a piece's macroharmony. The higher the degree of diatonicity of the macroharmony, the stronger the perception of tonality; the lower the degree of diatonicity the stronger the perception of atonality. Tonality and atonality are therefore no absolute entities but only extremities. Music is in most cases not either tonal or atonal,[141] or not even something in between, but music has a certain **degree of tonality (or atonality)** at any given moment in a piece's progress, which can be measured on the basis of its degree of diatonicity. The degree of tonality depends on how closely the macroharmony of a piece resembles or coincides with the tonal diatonic sets. A piece that uses all of the pitch classes of a tonal diatonic set and no others has the highest possible degree of tonality (its macroharmony is identical to one of the tonal diatonic sets). When on the other hand not all of the pitch classes of a tonal diatonic set are used (only a real subset of at least one of the tonal diatonic sets) or when other pitch classes occur (like chromatic alterations for instance) the degree of tonality of the piece decreases. Pieces that are constructed with pitch class sets

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[137] On necessary and sufficient features see: Scott Sturgeon, *Knowledge,* in A.C. Grayling (ed.), *Philosophy, A Guide through the Subject*, Oxford University Press, 1995, pp. 10-12.

[138] Reginald Smith Brindle, *Serial Composition*, Oxford University press, 1966, p. 11.

[139] Arnold Schönberg, *Harmonielehre*, Universal Edition, 1911. Translation by Roy E. Carter: *Theory of Harmony*, Faber & Faber, 1983, p. 432.

[140] Arnold Schönberg, *Harmonielehre*, p. 432.

[141] As will become clear later, a piece can be 'perfectly tonal' (have the highest possible degree of tonality), if it uses only one tonal diatonic set and if that set is sounded in its totality in the very first sound of the piece (see Section 3.5 on tonality analysis).

that differ considerably from the tonal diatonic sets have a low degree of tonality (or a high degree of atonality). The degree of tonality of a pitch class set is therefore determined by the degree to which it resembles the tonal diatonic sets, that is by its degree of diatonicity.

It is important to realise that the fact that there is a correlation between degree of tonality and degree of diatonicity is not to say that diatonicity and tonality are the same; it only means that degree of diatonicity is an appropriate measure for (the degree of) tonality. The higher the degree of diatonicity, the higher the degree of tonality will be, and the stronger all the features that come with it and that define tonality: the identification and perception of tonic, functionality and hierarchy. If, in addition, the conventional norms of common-practice are respected, the *perception* of tonality will be enhanced even further.

To make the distinction between degree of tonality and degree of diatonicity clear, compare the correlation between the two with the correlation between temperature and heat in physics. Diatonicity relates to tonality as temperature to heat. Although temperature and heat are correlated, they are not identical. "Temperature is a measure of the ability of randomly moving particles, usually atoms, to directly impart thermal energy[142] to a thermometer or any other object."[143] It is a measure of the average kinetic energy of the particles. Heat, on the other hand "is the thermal energy transferred, via atomic collisions, from a region of high temperature to a region of lower temperature."[144] It is often the case that when an amount of heat (measured in calories) is added to a substance, its temperature will rise, and vice versa. There is, in other words, usually[145] a correlation between the measurement of temperature change and heat. Heat is measured with a calorimeter consisting of an insulated container of water with a *thermometer*, and is measured as a relation to the change in *temperature* of that thermometer. The change in temperature indicates the heat (thermal energy transfer), even if heat and temperature are different qualities. Therefore temperature change is an appropriate way to measure heat, even though temperature and heat are different physical entities. The same can be said about diatonicity and tonality. The degree of diatonicity is an indication of the degree of tonality of a pc-set or piece of music, even though diatonicity and tonality are distinct properties of music. "We can exactly say what a calorie is, even without knowing what heat is."[146] Likewise, we can exactly say what the degree of tonality of a piece is, even without knowing what tonality is.

3.4 Quantification of tonality

3.4.1 Introduction

Music, it was said, is not either tonal or atonal but evolves between higher and lower degrees of tonality, for which the degree of diatonicity is an appropriate measure. This suggests that tonality is a measurable, a quantifiable feature. The aim of the present section is to show how degrees of tonality can be quantified, and to develop a 'tonality formula' that can be used to determine the degree of tonality of any pitch class set. It will lead to a classification of set classes according to their degree of tonality. This classification is only vaguely akin to the idea of "similarity index" developed by Robert Morris[147], to Ian Quinn's idea of "chord quality"[148], and to Dmitri Tymoczko's "musical distance"[149].

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[142] "Thermal energy is the net disordered kinetic energy […] associated with any group of particles, usually atoms of bulk matter" (Eugene Hecht, *Physics*, Brooks/Cole Publishing Company, 1994, p. 516 [in bold and italics in the source text]).
[143] Eugene Hecht, *Physics*, p. 517 [in bold and italics in the source text].
[144] Eugene Hecht, *Physics*, pp. 517-8 [in bold and italics in the source text].
[145] The addition of heat does not always cause a change in temperature. It may also result in a change of state of the material, such as melting or evaporation. It may be argued whether there is such a thing as 'change of state' in music. In other words, are there situations in which a rise of degree of diatonicity does not result in a (perception of) higher degree of tonality, or the other way around?
[146] Eugene Hecht, *Physics*, p. 518.
[147] See: Robert Morris, *A Similarity Index for Pitch-Class Sets*, Perspectives of New Music, Vol. 18, N°. 1/2, 1979-1980, pp. 445-60.

Robert Morris's approach is constructed around "a particular similarity relation based on the *interval-class-vector* (V) associated with the sets that are members of a SC [set class]. […] As with other similarity measures, the relation under consideration is generated by comparing the Vs of two sets and, by extension, their SCs. [In Morris's approach], however, the comparison is based on the total number of ics [interval classes] that are different: the less different the ics, the greater the similarity."[150] Also Ian Quinn's idea of "chord quality"[151] (relations of sonorities of pc-sets) relates to interval-class content of pitch class sets. Indeed, as Joseph N. Straus states: "the quality of a sonority can be roughly summarized by listing all the *intervals* it contains"[152]. Although, as will become clear later, interval-class content of pitch class sets or set classes (indicated by their interval-class-vectors) play a role in the quantification of consonance, they are not involved in the quantification of tonality. Dimitri Tymoczko's "three conceptions of musical distance"[153] are based on voice leading, acoustics and—once again—total interval content. As is the case with interval content, the concepts of voice leading and acoustics, too, are irrelevant for the idea of tonality developed in the present dissertation. Therefore, the present approach is considerably different from that of Robert Morris, Ian Quinn and Dmitri Tymoczko, in that it explores the relatedness of pitch class sets to the tonal diatonic sets, based on their being (similar to) subsets or supersets of the tonal diatonic sets.

An important preliminary remark on nomenclatura has to be made before starting the construction of the T-formula. The pitch class set theory developed by Allen Forte[154] does not distinguish between set classes and their inversion. Minor triads (with prime form (0,3,7)) and major triads (the inversions of minor triads, with prime form (0,4,7)), for instance, share the same forte number ([3-11]). Since the degree of tonality of a set class may differ from the degree of tonality of its inversion, different Forte names need to be introduced for inversions of set classes. This is done by adding "i" after the Forte name of the set class ([3-11] for the minor triads and [3-11i] for the major triads, for instance). Forte names will be further extended for single pitches ([1-1]), for interval-classes ([2-1] through [2-6]) and for set classes with 11 and 12 elements ([11-1] and [12-1]).

3.4.2 Construction of the T-formula

Being a subset of a tonal diatonic set is taken as the first criterion for similarity of a pitch class set (as a member of a set class) to the tonal diatonic sets. Therefore, the number of times the pc-set is a subset of the tonal diatonic sets is taken as a starting point in the construction of what will be called the tonality formula or **T-formula**, the formula to determine the **degree of tonality $T_{(c\text{-}n)}$** of a pitch class set with Forte name [c-n][155] for all possible pitch class sets. The number of times the pc-set is a subset of the tonal diatonic sets is indicated by the **S7-value ($S^7_{(c\text{-}n)}$)** of the pc-set (belonging to the set class) with Forte name [c-n]. The S7-value of a pc-set is the weighed sum (divided by 3)[156] of the number of times the set is a subset of each of the three tonal diatonic sets (tonal 7-sets). In symbols:

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148 See for instance: Ian Quinn, *A Unified Theory of Chord Quality in Equal Temperaments*, unpublished doctoral dissertation, 2004; Ian Quinn, *On Woolhouse's Interval-Cycle Proximity Hypothesis*, in Music Theory Spectrum, Vol. 32, N°. 2, 2010, pp. 172-9; or Ian Quinn, Listening to Similarity Relations, in Perspectives of New Music, Vol. 39, N°. 2, 2001, pp. 108-58. Ian Quinn, *General Equal-Tempered Harmony (Parts II and III),* Perspectives of New Music 45.1, 2007, pp. 4-63.

149 See for instance: Dmitri Tymoczko, *Three Conceptions of Musical Distance*, in: Elaine Chew, Adrian Childs, & Ching-Hua Chuan (eds.), *Mathematics and Computation in Music*, Springer, 2009, pp. 258-273; Dmitri Tymoczko, *Geometrical Methods in Recent Music Theory*, Music Theory Online 16.1, 2010, http www.mtosmt.org/issues/mto.10.16.1/ mto.10.16.1.tymoczko.html [last accessed: 24 January 2013].

150 Robert Morris, *A Similarity Index for Pitch-Class Sets*, p. 446 [italics added].

151 Ian Quinn, *A Unified Theory of Chord Quality in Equal Temperaments*, p. 12.

152 Joseph N. Strauss, *Introduction to Post-Tonal Theory*, Pearson Prentice Hall, 3rd edition, 2005, p. 11; also quoted in: Ian Quinn, *A Unified Theory of Chord Quality in Equal Temperaments*, p. 12 [italics added].

153 Dmitri Tymoczko, *Three Conceptions of Musical Distance*, pp. 258-73.

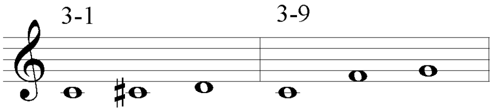
154 In: Allen Forte, *The Structure of Atonal Music*, Yale University Press, 1973.

155 "The number [c] to the left of the hyphen is the cardinal number of the set; the number [n] to the right of the hyphen is the ordinal number of the set—that is, the position of the prime form on the list [of set classes]" (Allen Forte, *The Structure of Atonal Music*, p. 12). As was mentioned before, this number is extended with "i" for the inversion of set classes.

156 This weighing factor is necessary to balance the combination of the S7-value with the S^{c-1} value added to the formula in the second step of the construction. The value was obtained in an experimental way.

$$S^{7}_{(c\text{-}n)} = (S^{7\text{-}32}_{(c\text{-}n)} + S^{7\text{-}34}_{(c\text{-}n)} + S^{7\text{-}35}_{(c\text{-}n)}) / 3 \qquad \text{(Formula 1)}$$

In this formula $S^{7\text{-}32}_{(c\text{-}n)}$ stands for the number of times pc-set [c-n] is a subset of pc-sets of set class [7-32]. Mutatis mutandis, the same applies to $S^{7\text{-}34}_{(c\text{-}n)}$ and $S^{7\text{-}35}_{(c\text{-}n)}$. Take for instance the pc-sets with Forte names [3-1] and [3-9] (Example 3.12)[157]:



Example 3.12: Prime form representations of set classes [3-1] and [3-9].

[3-1] does not occur in any of the tonal 7-sets; it is no subset of [7-35], [7-34] or [7-32], so it will have a very low degree of tonality. [3-9] on the other hand occurs five times in [7-35] (the major scale, natural minor scale or the church modes), three times in [7-34] (the melodic minor scale) and two times in [7-32] (the harmonic minor scale). In total, [3-9] is 10 times a subset of the tonal 7-sets, [3-1] is no subset. [3-9] appears to be a set class with a very high degree of tonality (within the group of set classes with cardinality 3 at least). In symbolic notation:

$$S^{7}_{(3\text{-}1)} = 0 \qquad\qquad S^{7}_{(3\text{-}9)} = 10 / 3$$

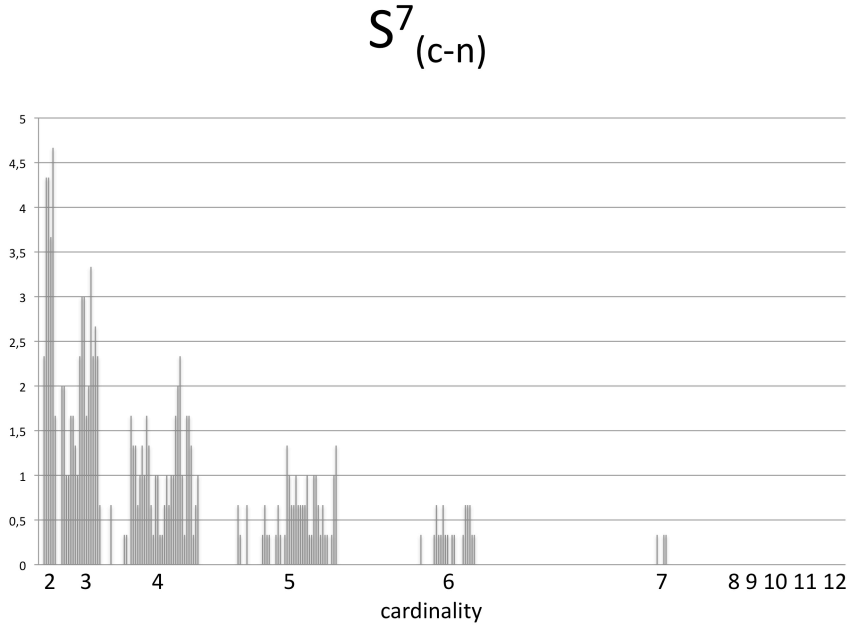
This means that pieces consisting entirely of the pitch classes of a pc-set [3-9] will be called ‘more tonal’ (or ‘less atonal’) than pieces using the pitch classes of [3-1]. In fact, [3-9] has the highest S7-value of all set classes with three elements (see Example 3.13 below). As a result, it will show to have a high degree of tonality. This is consistent with the correspondence between tonality and functionality. Indeed, [3-9] contains the 1^{st}, 4^{th} and 5^{th} degree of the tonal 7-sets, Riemann’s tonic, subdominant and dominant.

pc-set (3-n)	$\mathbf{S^{7}_{(3\text{-}n)}}$
3-1	0
3-2	2
3-2i	2
3-3	1
3-3i	1
3-4	1,66
3-4i	1,66
3-5	1,33
3-5i	1
3-6	2,33
3-7	3
3-7i	3
3-8	1,66
3-8i	2
3-9	3,33
3-10	2,33
3-11	2,66
3-11i	2,33
3-12	0,66

Example 3.13: S7-values for pc-sets with three elements.

[157] The pitch class sets in the examples are not (always) written in prime form but have sometimes been transposed (for reasons of convenience) in such a way that the resemblance with the major or minor scales on C is clear.

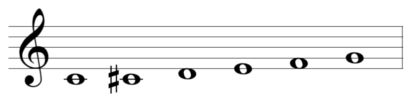
Example 3.14 below shows the S7-value for all set classes calculated with Formula 1 (the set classes are grouped according to their cardinality (their number of elements) on the horizontal axis):



Example 3.14: S7-values for all set classes.

Although it doesn't show precise S-7 values, it should be clear from Example 3.14 that the S7-value of a pc-set or set classes is not sufficient to determine its degree of tonality. First of all, it only works well for pc-sets with only three or four elements, but it is inappropriate for pc-sets with a higher cardinality. Pc-sets with five or six elements, for instance, are rarely subsets of the tonal 7-sets, even if they contain only one pitch class that does *not* belong to the tonal 7-sets and should therefore be considered highly tonal (because they resemble the tonal 7-sets very much); and it wouldn't apply at all to pc-sets with eight elements or more (because they are never a subset of the tonal 7-sets), as can be seen in Example 3.14.

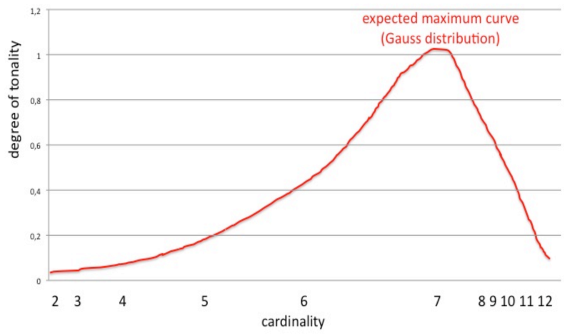
To explain the reason for this problem in detail, take pc-sets of set class [6-z11] for instance. If the C-sharp were left out in Example 3.15 representing an instance of [6-z11], this would change [6-z11] into [5-23], which is supposed to have a high degree of tonality (at least, it has a relatively high S7-value)[158].



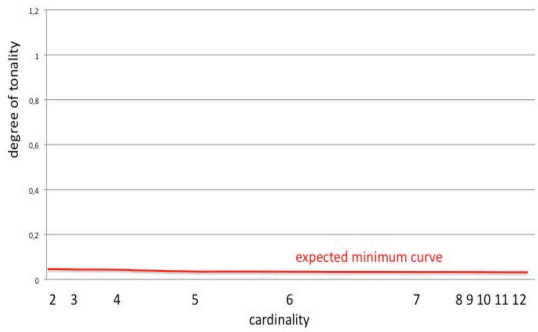
Example 3.15: Prime form representation of set class [6-z11].

If similarity with the tonal 7-sets is the criterion that determines the degree of tonality of a pc-set or set class, one would expect the maximum and minimum values of the degree of tonality within a cardinality group (a group consisting of set classes with the same number of elements) to evolve roughly according to the curves in Example 3.16 a and b below. The maximum values would resemble a Gauss distribution.

[158] [5-23] is a subset of [7-32] and [7-34], and twice a subset of [7-35], which is much for a set class of cardinality 5.

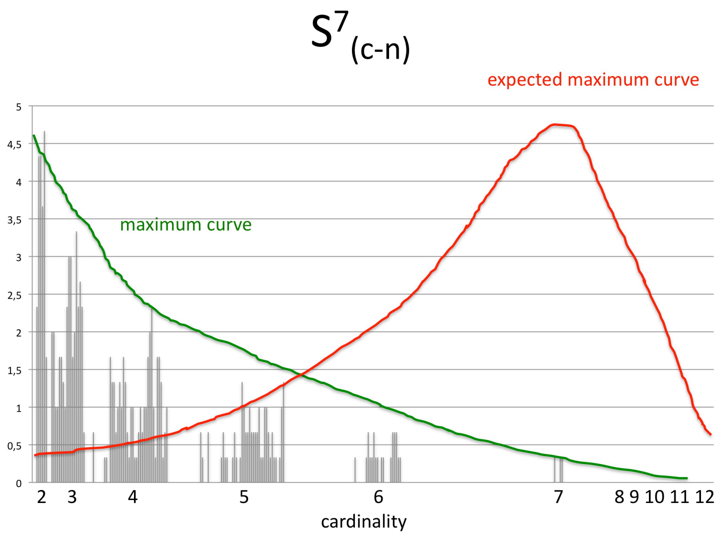


Example 3.16 a: Expected maximum curve.



Example 3.16 b: Expected minimum curve.

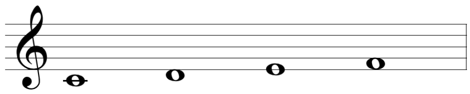
Example 3.14 has a maximum curve that is very different from the expected curve, as is shown in Example 3.17.



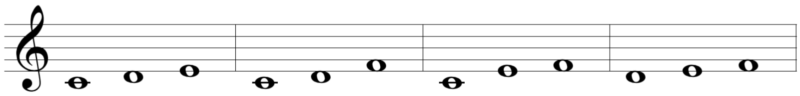
Example 3.17: Maximum curve for $S^7_{(c-n)}$, and expected maximum curve for $T_{(c-n)}$.

Therefore, counting subsets of the tonal 7-sets is not enough. The S7-value of the subsets of the set class under investigation that contain one pitch class less should also be taken into account to determine a degree of tonality that corresponds better with the diatonicity-based definition of tonality (and with the reality of perception), and to provide a solution to the problem of set classes with more than seven elements.

To illustrate this second step in the construction of the T-formula, consider set class [4-11i] in Example 3.18). This pc-set is only 4 times a subset of the tonal 7-sets ($S^7_{(4\text{-}11i)}$ = 1,33), although it resembles [7-35] more than do all of its subsets with one element less (Example 3.19), that have higher S7-values, as the table in Example 3.20 shows.



Example 3.18: A pc-set [4-11i].



Example 3.19: Subsets of a [4-11i] with cardinality 3.

pc-set	$\mathbf{S^7_{(c\text{-}n)}}$
4-11i	1,33
3-2i	2
3-4i	1,66
3-6	2,33
3-7	3

Example 3.20: S7-values for [4-11i] and its subsets with one element less.

The average S7-value for all subsets of [4-11i] with cardinality 3 (one element less) should therefore also be taken into account to determine the degree of tonality of [4-11i]. This average is calculated as follows on basis of the values in the table in Example 3.20: (2 + 1,66 + 2,33 +3) / 4 = 2,25. In general, this average is represented in symbols as follows:

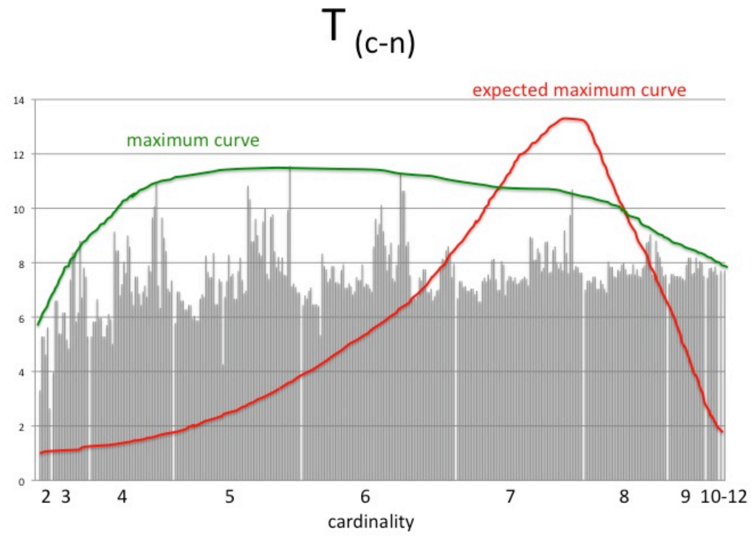
$$[\sum(S^{c-1}_{(c\text{-}n)})]/c \qquad \text{(Formula 2)}$$

$S^{c-1}_{(c\text{-}n)}$ is the S7-value for a subset of [c-n] with one element less (c still stands for the cardinality of set class [c-n]). This approach affects the value of the degree of tonality $T_{(c\text{-}n)}$ of any pc-set in a recursive way. The starting point for the recursion is established at $\sum(S^1_{(2\text{-}n)}) = 1$, which would mean the degree of tonality of set class [1-1] is not zero. This is in accordance with Schoenberg's idea, mentioned above, that absolute atonality does not exist in pitched music. In other words, the degree of tonality of a pitched sound is always higher than zero. $T_{(c\text{-}n)} = 0$ only occurs if there is no tone, no pitched sound.

Adding the new element S^{c-1}, a provisional tonality formula can be written as a combination of Formulas 1 and 2:

$$T_{(c\text{-}n)} = S^7_{(c\text{-}n)} + [\sum(S^{c-1}_{(c\text{-}n)})]/c \qquad \text{(Formula 3)}$$

The provisional degree of tonality values obtained with this formula are shown in Example 3.21.



Example 3.21: Degree of tonality for all set classes with Formula 3.

The maximum curve looks a little more like the expected maximum curve and the values for set classes with cardinality higher than 7 are no longer zero, but still the results do not correspond with the idea that degrees of tonality should increase when cardinalities approach seven. To obtain this, a Gauss factor $G_{(c)}$ is introduced, which corresponds to the fact that the closer the number of elements in a pc-set is to 7, the higher its degree of tonality. $G_{(c)}$ is determined with the following formula for Gauss distributions:

$$G(x)=1+3e^{-\frac{1}{4,5}x^2+\frac{14}{4,5}x-\frac{49}{4,5}} \qquad \text{(Formula 4)}^{159}$$

c	G(c)
2	1,01
3	1,09
4	1,41
5	2,23
6	3,40
7	4,00
8	3,40
9	2,23
10	1,41
11	1,09
12	1,01

Example 3.22: Values of the Gauss factor for every cardinality (c).

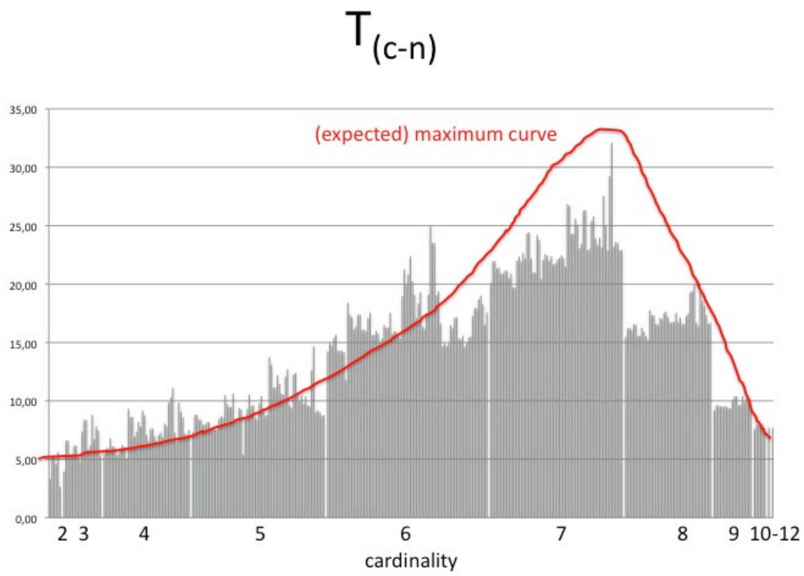
[159] The Gauss-distribution function is of the form:

$$G(x)=e^{\left(\frac{-x^2+2\mu x-\mu^2}{2\sigma^2}\right)}$$

where variance σ = 1,5 (value obtained experimentally) and mean (the location of the peak) μ = 7.

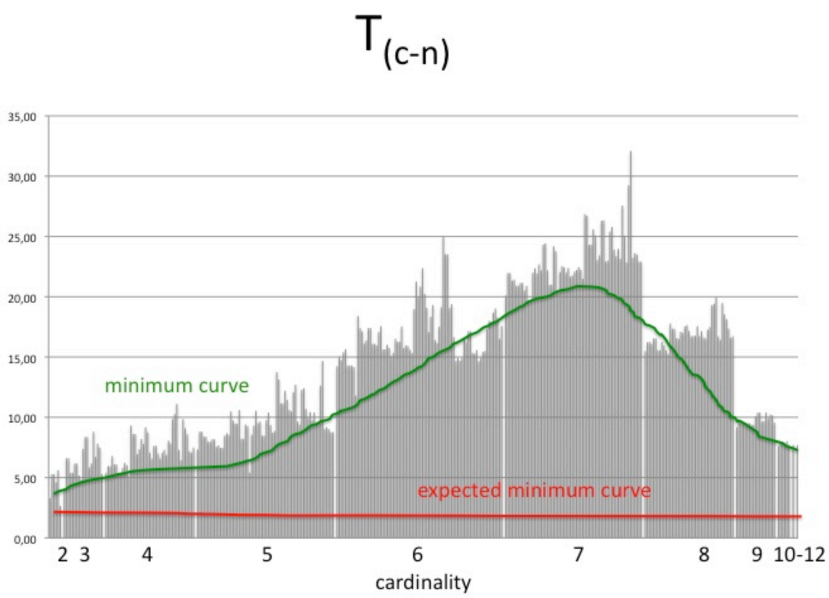
The provisional $T_{(c\text{-}n)}$ values obtained with Formula 3 are multiplied by the Gauss factor corresponding to the cardinality of the set class (listed in the table of Example 3.22 above). This results in Formula 5, which yields the values shown in Example 3.23 below.

$$T_{(c\text{-}n)} = G_{(c)} \cdot (S^7_{(c\text{-}n)} + [\sum(S^{c-1}_{(c\text{-}n)})]/c) \qquad \text{(Formula 5)}$$



Example 3.23: Degree of tonality for all pc-sets obtained with Formula 5.

Formula 5 approaches the expected maximum curve sufficiently, as can be seen in Example 3.23. The minimum curve, however, is still far from the expected minimum curve, as can be seen in Example 3.24.



Example 3.24: Minimum curve resulting from formula 5,
and expected minimum curve.

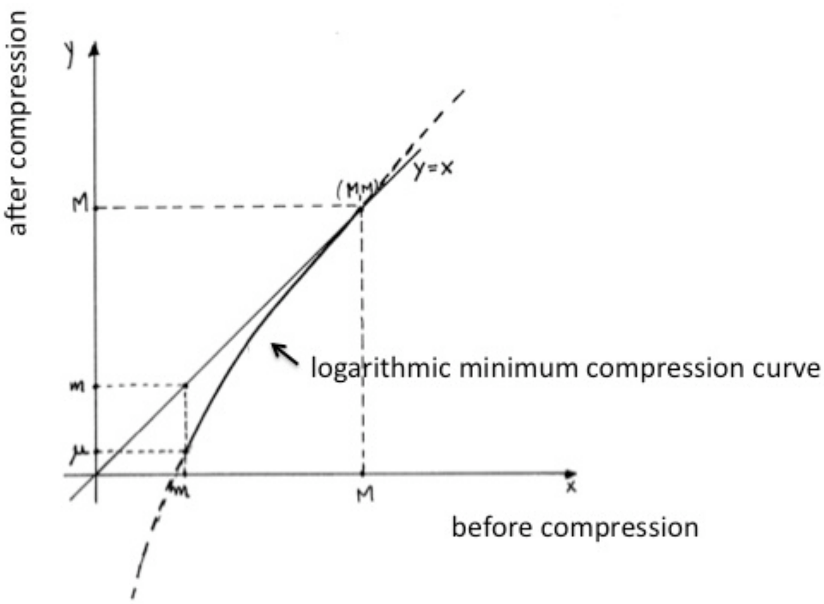
Also, the degree of tonality has a different value for the three tonal diatonic sets. There is no reason, however, to postulate that one tonal 7-set is 'more tonal' than another. Therefore, in a next step, the formula has to be adapted in order to produce the same $T_{(c\text{-}n)}$ value for [7-32], [7-34] and [7-35]. This is done by adding weighing factors to Formula 1, as shown in Formula 6. This procedure has no significant effect on the relative degree of tonality other pitch class sets.

$$S^{7}_{(c\text{-}n)} = (1{,}68 \,.\, S^{7\text{-}32}_{(c\text{-}n)} + 1{,}32 \,.\, S^{7\text{-}34}_{(c\text{-}n)} + 0{,}52 \,.\, S^{7\text{-}35}_{(c\text{-}n)}) / 3 \qquad \text{(Formula 6)}$$

A last adaptation has to be made to keeps the maximum values the way they are, while lowering the other values proportionally in order to bring the minima to the expected values. The best result is obtained with the following logarithmic minimum compression function (Formula 7):

$$\mathcal{K}(x) = x + [(\mu\text{-}m) / \mathit{ln}(m/M)] \,.\, \mathit{ln}(x/M) \qquad \text{(Formula 7)}$$

The curve for this function is shown in Example 3.25.



Example 3.25: Logarithmic minimum compression curve.

In formula 7, M stands for the maximum value for the degree of tonality within the cardinality group of the pc-set in question; m is the minimum value; μ is the minimum value after compression of m. The value for μ depends on the cardinality of the set class and is determined with Formula 8 below.

$$\mu = M / (c \,.\, G_{(c)}) \qquad \text{(Formula 8)}$$

Example 3.26 lists the values of M, m and μ for cardinalities 2 to 12.

c	M	m	μ
2	6,26	3,18	3,10
3	9,99	4,89	3,06
4	15,83	8,15	2,81
5	26,02	14,78	2,33
6	38,78	23,31	1,90
7	44,48	30,84	1,59
8	33,95	27,31	1,25
9	20,71	18,57	1,03
10	12,72	12,12	0,90
11	9,67	9,67	0,81
12	8,87	8,87	0,74

Example 3.26: Values for M, m and μ for all cardinalities (c).

Applying the logarithmic minimum compression function to Formula 5 results in the following formula (Formula 9):

$$T_{(c\text{-}n)} = \mathcal{K}[G_{(c)} \,.\, (S^{7}_{(c\text{-}n)} + [\sum(S^{c-1}_{(c\text{-}n)})]/c)] \qquad \text{(Formula 9)}$$[160]

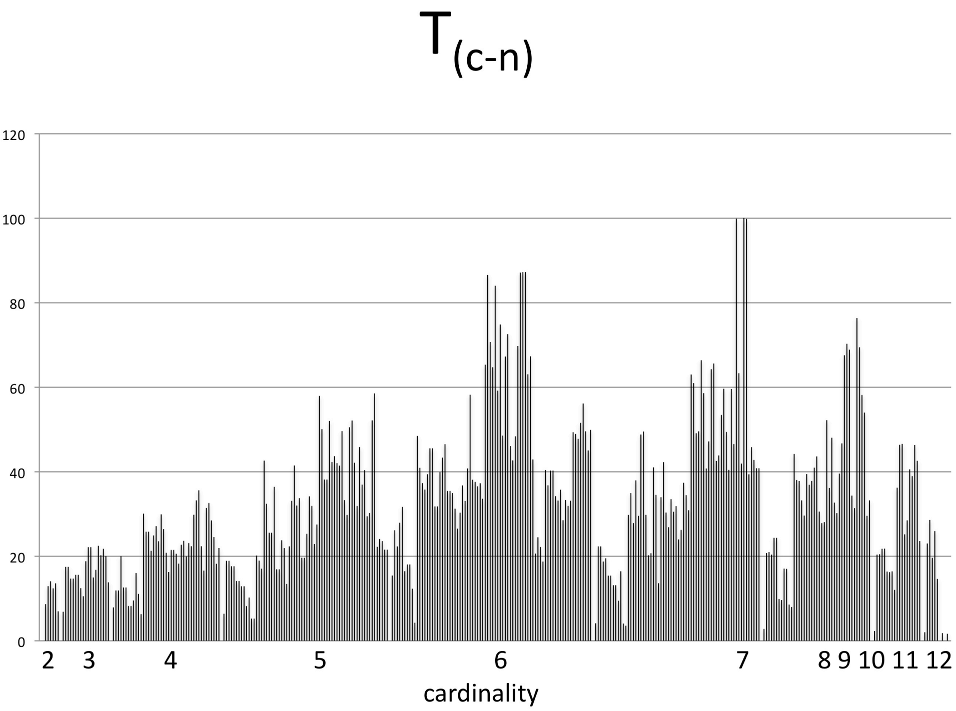
Finally, for reasons of elegance, every value of $T_{(c\text{-}n)}$ is multiplied by 2,25 to obtain degrees of tonality of 100 for the three tonal 7-sets.[161] This results in the final T-formula (Formula 10).

T-Formula:

$$\boxed{T_{(c\text{-}n)} = 2{,}25.\mathcal{K}[G_{(c)} \,.\, (S^{7}_{(c\text{-}n)} + [\sum(S^{c-1}_{(c\text{-}n)})]/c)]} \qquad \text{(Formula 10)}$$

The values of the degree of tonality obtained with the T-formula are shown in Example 3.27 below. Exact $T_{(c\text{-}n)}$ values for all pitch class sets are listed in Appendix 1.

[160] For simplicity reasons, the weighing factors are not included in the representation of the average of the sums of S^{c-1} values.
[161] $T_{(c\text{-}n)}$ values are no percentage value. A $T_{(c\text{-}n)}$ value of 60 does not mean the pitch class set (c-n) has a degree of tonality of 60%. It is just a relative value—without unit—between 1 and 100, 1 being the lowest possible degree of tonality, 100 the highest.



Example 3.27: Degree of tonality for all set classes
obtained with the T- Formula (Formula 10).

3.5 Application of the T-formula in analysis

Although the T-formula is first and foremost constructed to assess the technique of CIG-serialism, it can also be used as an analytical tool. Indeed, the formula not only allows for the determination of the degree of tonality of any given pitch class set, but also of the instantaneous and average degree of tonality of any piece of music that is based on the pitch classes of the chromatic scale. The tonality analysis of such pieces gives an idea of how (the perception of) the degree of tonality changes in the course of a piece. The **instantaneous degree of tonality** of the majority of pieces (the degree of tonality at a specific moment of a piece's progress) evolves during the course of the piece. Most pieces from before the 20th century have a high degree of tonality very soon after they start and at their very end, but what happens in between depends on the piece, the personal idiom of the composer, and the period in history the piece was written in. Music that is generally called tonal will show to have a high **average degree of tonality** when analysed with the T-formula, and what is generally called atonal music will show to have a low average degree of tonality. Therefore, although the approach with the tonality formula is different from traditional approaches (which do not use quantification) the result is similar and in accordance with general perception.

The present section describes the **Tonality Analysis Technique** or **TA-technique**, which is developed to determine the average and instantaneous degree of tonality and to visualise the evolution of the instantaneous degree of tonality within the course of a piece. As a first step and basic idea in the TA-technique, the piece has to be divided into pc-set 'sections' chosen in such a way that they all have the highest possible degree of tonality. This approach can be justified by the fact that tonally acculturated listeners automatically and in many cases unconsciously interpret music according to a tonal framework, as was discussed in section 3.3. Whenever there is a possibility for tonally acculturated listeners to discern tonality in (a section of) a piece, they will perceive tonality. A short section with a relatively high degree of tonality in a highly atonal piece (such as the elements that had to be avoided in atonal series, according to Reginald Smith Brindle) will be perceived as belonging to a tonal idiom.

The TA-technique always operates backwards in time. This means that when the degree of tonality of a piece is determined at a certain moment of its progress, the pc-set section that determines the degree of tonality can only contain notes that have already sounded, with the moment for which the instantaneous degree of tonality is measured at the very end of the section. Although listeners have expectations about the further course of a piece (certainly when it belongs to common-practice)[162], they can never be sure about the future development of a piece when they hear it for the first time. The degree of tonality can therefore only be determined on the basis of the information that is already conveyed to the listeners, as will become clear in the examples below. This renders the technique for tonality analysis developed here more in accordance with actual perception than the classical methods of functional harmonic analysis. In these classical methods, the very first chord of a piece is generally attributed a function (tonic in most cases), although in fact a listener cannot determine the function of this chord without knowledge of what still has to come. Indeed, on a purely perceptual level it is impossible for the listener to determine whether a piece is highly tonal or not after hearing the first note or chord.

In traditional harmonic analysis, the G on the first beat of the *Aria* from Johann Sebastian Bach's *Goldberg Variations* BWV 988 (from 1741), for instance, is generally analysed as a the root of the chord of G major with a tonic function (see Example 3.28). This, however, is only a theoretical statement. On hearing the notes of the first beat of the piece, a listener cannot possibly decide whether the note heard belongs to a piece in G major, and even less what its tonal function in the piece is.

Example 3.28: Johann Sebastian Bach, *Aria* from *Goldberg Variations* BWV 988, bar 1-7.

If the listeners know that the piece is by Bach or that it belongs to the tonal tradition of common-practice music, they will expect it to be tonal, but they cannot conclude this from the perceptual information provided by the first beat. A single starting pitch class can belong to a highly tonal piece as well as to a highly atonal piece. Therefore, on the basis of perception, the listeners have to postpone their judgment about the degree of tonality of the piece (and the tonal function of the sounds heard). The degree of tonality is therefore $T_{(1\text{-}1)} = 1$ on the first beat of Bach's *Aria*.

[162] See David Huron, *Sweet Anticipation: Music and the Psychology of Expectation*, MIT press, 2006.

The onset of a piece has often got a low degree of tonality. The expectation[163] of the tonally acculturated listener 'predicts' tonality however. As David Huron says: "Over the eons, brains have evolved a number of mechanisms for predicting the future."[164]. This expectation may today be completely wrong.[165] Kofi Agawu puts it like this:

> Beethoven's Fifth Symphony begins with a four-note motive containing two pitch classes, G and Eb.[166] In the universe of major and minor scales (including melodic, harmonic, and natural minor as distinct constructs) these two pitch classes belong to three major keys and eleven minor keys. You could say, therefore, that the opening is ambiguous in the sense that it gives rise to two or more harmonic meanings. It would be an extraordinary listener, however, who claimed to hear simultaneously fourteen different harmonic meanings at the beginning of Beethoven's Fifth. Once a context is taken into account – and by 'context' I mean a series of additional texts – ambiguity dissolves into clarity. After all, this is a work in C minor, its opening plays with and against the Classical convention of beginning, the four-note motive is sequentially repeated on pitch classes F and D, and so on: these kinds of observation serve to eliminate most if not all of the alternative meanings.[167]

The classical methods of tonal harmonic analysis are therefore highly theoretical, whereas the TA-technique is more in accordance with actual perception. The following examples serve to illustrate the ideas and implementation of the TA-technique in practice.

3.5.1 Highly diatonic music

Let us return to the *Aria* from Bach's *Goldberg Variations* first (see Example 3.28). As was mentioned, the piece starts with a single pitch class (G), with $T_{(1\text{-}1)} = 1$. On the second beat a B is added. The piece consists of a pitch class set (of set class) [2-4] now (an instance of ic 4 consisting of G and B)[168]. The degree of tonality of the piece at this stage rises to $T_{(2\text{-}4)} = 12$, and so do the expectations of the listener that the piece will prove to be highly tonal.

On the last beat of bar 1, pitch classes A and D are heard for the first time. Although G and B do not sound at that moment, the listener still remembers them and constructs a Gestalt[169] containing the four pitch classes G, B, A and D. This is an instance of [4-22], with a degree of tonality $T_{(4\text{-}22)} = 30$. Similarly, in bar 2 and 3, F sharp, E and finally C sharp are added, respectively changing the pitch class set of the piece into a [5-27], [6-32] and [7-35] (see Example 3.29), and further raising the degree

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[163] On expectancy: see Douglas A. Bernstein, Edward J. Roy, Thomas K. Srull, Christopher D. Wickens, *Psychology*, Houghton Mifflin Company, 2nd edition, 1991, p. 199.

[164] David Huron, *Sweet Anticipation*, p. 5.

[165] Stockhausen's klavierstück X, for instance, begins with a major third (F-A), suggesting a tonic in F major, but continues in a highly atonal way. Schoenberg's second piece of the *Sechs Kleine Klavierstücke* op. 19 similarly starts on a repeated major third (G-B) which is even extended to a perceived major triad on G with the added leading not (F sharp) at the end of the second bar, but the degree of tonality never gets higher than an exceptional peek of 63 ($T_{(7\text{-}32i)}$) in the third bar, the average degree of tonality being a modest $T_{av} = 34$.

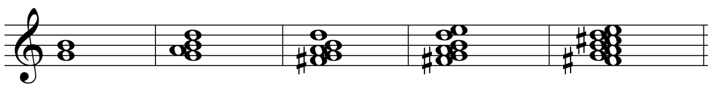
[166] See Example 1.10 c.

[167] Kofi Agawu, *Ambiguity in Tonal Music: A Preliminary Study*, in: Anthony Pople (ed.), *Theory, Analysis and Meaning in Music*, Cambridge University Press; New Ed edition (2 Nov 2006), p. 86.

[168] Note that [2-4] is not a standard Forte number for interval class 4 (ic 4), as was the case for [1-1], which is not a Forte number to indicate a single pitch class.

[169] The concept of Gestalt is here defined as a form or shape that has a meaning that is broader than and different from the summed meaning of its constituents (e.g. through the emergence of functionality, hierarchy, etc). The aesthetic idea for which an artwork is the sign vehicle (see Part 2) can also be seen as the idea of a Gestalt, an idea that represents its whole meaning, an "idea of something in total—of a Gestalt of a whole piece" (Felix Greissle, quoted in Joan Allen Smith (ed.), *Schoenberg's Way*, in Perspectives of New Music, Vol. 18, N°1/2 (Autumn, 1979 - Summer, 1980), p. 262).

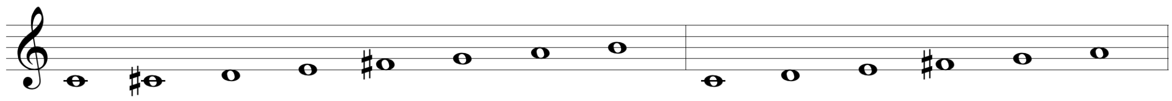
of tonality to $T_{(5\text{-}27)} = 41$, $T_{(6\text{-}32)} = 87$, and finally $T_{(7\text{-}35)} = 100$. At this point, the piece is completely diatonic and has reached the highest possible degree of tonality.



Example 3.29: The pc-sets in bar 1-3 of the *Aria* of the *Goldberg Variations*.

Surprisingly, the tonal diatonic set reached at this point corresponds to D major, and not—as the key signature and a possible perception of key would suggest—G major. This, however, is not an issue, since the TA-technique is only intended to indicate *how tonal* a piece is at a certain moment of its progress, not *which key* it is in. It should also be mentioned, that—based on the sonic elements present—the *Aria* could still be perceived in D major, starting on a IV (subdominant on G) in the first bar, moving to a tonic in bar 2 and then to a dominant (diminished seventh chord consisting of C sharp, E and G), but the first bar will steer most people's perception in the direction of G major. If the piece is stopped on the second beat of bar 3, this can be perceived as a conclusion, and then the piece would be in D major; it would have (imperceptibly) 'modulated'[170] from G to D, or the listener could reinterpret the first chord as a subdominant in hindsight. Anyway, no matter which key is perceived, the piece is perceived as being 'purely tonal'[171].

When C natural is added at the very end of bar 4, the perception can go different ways and so can the perceived degree of tonality. C sharp can be added to the [7-35] set reached so far, or one can leave C sharp out of the pitch class set and replace it by C natural to form a new Gestalt from the beginning of bar 4 (after the C sharp has stopped at the end of bar 3), but then B has to be left out as well, because there is no B in bar 4. So either the piece is perceived to be based on the pitch class set containing C, C sharp, D, E, F sharp, G, A and B (an instance of [8-23]) or (if C sharp is left out) on the set containing C, D, E, F sharp, G, and A, which is an instance of [6-33i], as is shown in Examples 3.30 a and b. The degree of tonality at this point would then be either $T_{(8\text{-}23)} = 69$ or $T_{(6\text{-}33i)} = 87$. Other strategies could also be considered, for instance making a tabula rasa and starting a whole new pitch class set containing only C, D and A (the notes sounding on the last quaver of bar 3). However, this is not what tonally acculturated listeners would do in their (unconscious) judgment.[172] Their cultural conditioning inevitably makes them choose for the interpretation that yields the highest possible degree of tonality (in this case: $T_{(6\text{-}33i)} = 87$).



Example 3.30 a & b: Two possible perceived pitch class sets
at the end of bar 4 of the *Aria* of the *Goldberg Variations*.

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170 I put 'modulated' between quotation marks, because no actual modulation happened. A modulation would suppose a replacement of one pitch class in the piece's set by another (this includes transient modulations and momentary tonicization (see Robert Gauldin, *Harmonic Practice in Tonal Music*, W.W. Norton & C[ie], 2[nd] edition, 2004, pp. 366 & 387) which is not the case here.

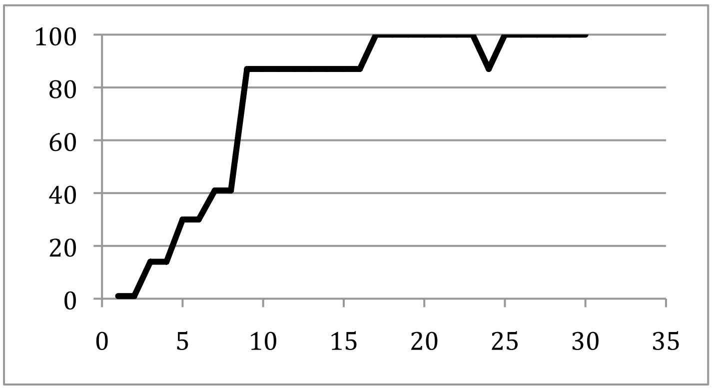
171 Purely tonal means: with the highest possible degree of tonality (T = 100).

172 As my friend and composer Luc Brewaeys claims: "I think most everybody who has been brought up within the Western musical tradition would inevitably look for tonal links in whatever type of music, old or new. I even think that this happens unconsciously" ("ik denk dat zowat iedereen die grootgebracht is geweest in de westerse muziektraditie sowieso tonale verbanden probeert te zoeken in welke muziek dan ook, oude of nieuwe. Ik denk zelfs dat het ook onbewust gebeurt." Luc Brewaeys in an e-mail to me on 26-03-2011 [my translation]). Composer and conductor Daan Janssens suggested that those links could be modal as well as tonal, but this makes no difference in my method of tonal analysis (Daan Janssens in a mail to me on 29-03-2011).

At the beginning of bar 5, a B is heard, which turns [6-33i] again into [7-35]. The degree of tonality rises again to $T_{(7\text{-}35)} = 100$. The piece is indubitably in G major at this point. Even when F natural is added at the end of bar 5, all the pc's of [7-35] (C, D, E, F, G, A, B) have sounded since the last appearance of F sharp, so the (perception of) degree of tonality is not affected by the occurrence of the chromatic change and the degree of tonality remains 100.

As a general rule the instantaneous degree of tonality of a piece at a certain moment can be determined by considering all the pitch classes that sound at that moment, and then going back in time, adding all the pitch classes that occurred before that moment as long as the degree of tonality can be made higher. This corresponds to how tonally acculturated listeners (unconsciously) interpret the degree of tonality of a piece they listen to. They do have expectations of what lies ahead in time, but they can only judged what they have heard, and they try to interpret music 'as tonally as possible', as was discussed before. When there are different possible interpretations, the tonally acculturated listener involuntarily chooses the most tonal one.

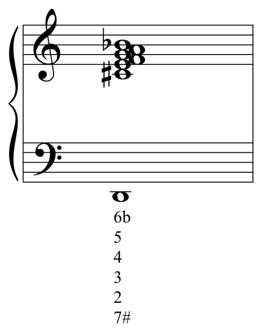
The evolution of the instantaneous degree of tonality of a piece can be represented in a graph such as the one in Example 3.31 below, which shows the evolution of the **T-curve** (a curve showing the evolution of instantaneous degree of tonality of a piece) for the first five bars of the *Aria* of Bach's *Goldberg Variations*. The unit of the horizontal axis in this T-graph is a quaver (there are 6 quavers to a bar). The graph shows how the T-curve reaches the maximum (100) on the 17th quaver of the piece (when C sharp is heard for the first time, see Example 3.28) and stays there, apart from a very small drop in the degree of tonality on quaver 24 (caused by C natural at the end of bar 4).



Example 3.31: T-curve for the first five bars of the *Aria* of the *Goldberg Variations* (unit of the horizontal axis = quaver).

This is a typical T-curve for the onset of a highly diatonic piece belonging to the common-practice. The maximum degree of tonality is gradually reached, generally within the span of a couple of bars. The *Vorspiel* to Wagner's *Das Rheingold*, is a rare exception to this rule because in it the highest degree of tonality is only reached after 129 bars. The first four bars contain only E flat. B flat is added in bar 5, suggesting a tonic-dominant relation to the tonally acculturated listener, although the degree of tonality is only $T_{(2\text{-}5)} = 14$ at that moment. It takes another 13 bars before a next pitch class (G) is added (in bar 18) to form the major triad of E flat. The first 48 bars of this piece are completely based on that triad (set class [3-11i] in inversion, which has a value $T_{(3\text{-}11i)} = 20$). In bar 49 and 50, two pitch classes are added to the piece (F and A-flat), thus getting closer to the scale of E-flat major forming an instance of set class [5-23] (the first five notes of the major tonal scale). The degree of tonality increases to $T_{(5\text{-}23)} = 58$ at this point. In bar 129, the scale of E-flat major ([7-35]) has finally occurred in its entirety when C is added, and the piece reaches the highest possible degree of tonality ($T_{(7\text{-}35)} = 100$). So, while, according to the functionality-definitions of tonality, it is impossible to determine whether the first 136 bars of this *Vorspiel* are tonal because they only feature one chord (one tonal function), with the TA-technique it is possible to describe an evolution in the degree of tonality of the piece; an evolution that is in conformity with a perceived sense of tonality (most people will have determined that the piece is tonal by bar 136, and will perceive the E-flat major triad as the tonic in E-flat major at this point).

On the opposite extreme of such a slow build-up towards the highest degree of tonality, tonal pieces starting with the highest degree of tonality (with a T-curve starting on 100) are also rare. This happens only if a complete tonal 7-set is used from the very start of the piece. A rare (and early) example of such pieces is the ballet *Les Elémens* (from 1738) by Jean-Fery Rebel[173]. The first notes of *Le Cahos*—the first movement of the ballet (see Example 3.32)—consist of a chord containing D, E, F, G, A, Bb and C# (an instance of [7-32]).



Example 3.32: Jean-Fery Rebel, *Les Elémens*, opening chord.

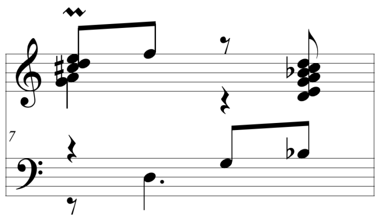
The opening bars of Richard Strauss' *Alpensymphonie* constitute a remarkable example of how all the pitch classes of a [7-35] set ($T_{(7\text{-}35)} = 100$) are not only introduced one by one, but also sustained in order to sound as a complete set from bar 3 on (see Example 3.33)[174].

[173] This example was kindly suggested to me by dr. Luk Vaes. The example features in his doctoral dissertation: Luk Vaes, *Extended Piano Techniques In Theory, History and Performance Practice*, unpublished, 2009, pp. 92-3.
[174] The *Augurs chord* with which the *Augurs of Spring* of Stravinsky's *Sacre du Printemps* starts is another example of a complete tonal 7-set ([7-32] in this case) right from the start of (a movement in) a piece of music.

Example 3.33: Richard Strauss, *Alpensymphonie* bar 1-12.

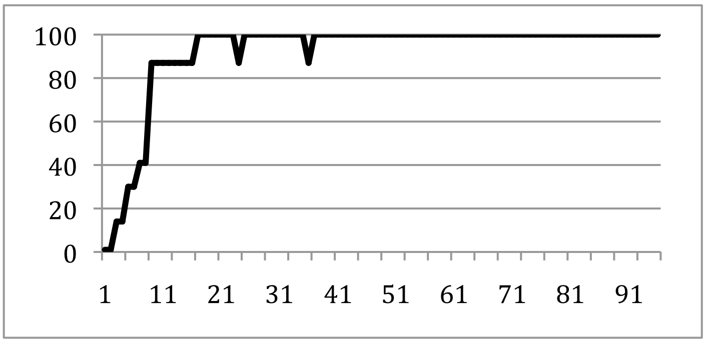
The simultanious presence of many or all the pitch classes of a tonal diatonic set may lead to highly dissonant music, but it does not result in atonality.[175] In Domenico Scarlatti's *Sonata in a minor* K175, which is a "study in the playing of acciaccaturas […] [m]any of the chords contain up to ten notes, including some outside accepted hamony"[176]. This results in dissonant sound combinations, but not in highly atonal music as long as the notes in the chords belong to one of the tonal diatonic sets or to a set very similar to a tonal diatonic set, even if they do not belong to "accepted harmony". This is illustrated in bar 25 of the sonata, shown in Example 3.34. The first and last quavers of this bar consist of chords containing six different pitch classes (if the acciaccatura F on the first chord is taken into account). The pc-sets formed by those pitch classes are subsets of [7-32] (harmonic D minor): [6-z24] and [6-z29] respectively. The degree of tonality of both sets is high ($T_{(6\text{-}z24)} = 87$ and $T_{(6\text{-}z29)} = 73$). The chords are therefore highly tonal. This would even be the case if the complete tonal diatonic set [7-35] would be sounded simultaniously. As was discussed before, such a chord may even be analysed functionally; it can be interpreted as a dominant 13th chord (see Example 3.11).

[175] The distinction between tonality/atonality and consonance/dissonance will be discussed in the chapter on consonance.
[176] Liner notes to: Scott Ross, *Domenico Scarlatti, The Complete Keyboard Sonatas*. (34 CD's) Warner Classics, 2005, p. 94.



Example 3.34: Bar 25 of Domenico Scarlatti's *Sonata in a minor* K175.

Let us now return once again to the *Aria* from the *Goldberg Variations*. For the first five bars analysed above, the average degree of tonality is $T_{(average)} = 75,16$. The average degree of tonality of a piece (or a section of a piece in this case) is calculated by adding the degree of tonality of all 'time units' in the piece (the units on the horizontal axis of the T-curve, quavers in this case) and dividing this sum by the total number of units. The average for the first five bars of the Aria is relatively low, because the section is taken from the start of the piece and it takes 17 quavers to reach the maximum degree of tonality. The average would only rise if it were calculated over a longer time span of any predominantly diatonic piece, or if the beginning of such a piece were left out of the picture. In the case of Bach's *Aria*, the average degree of tonality for the first sixteen bars attains the very high value $T_{(average)} = 92,10$, because from bar 7 on, the degree of tonality stays at 100, as can be seen from Example 3.35below. Without the first seventeen quavers of the piece, the average is even $T_{(average)} = 99,67$; almost purely diatonic.



Example 3.35: T-curve for the first 16 bars of the *Aria* of the *Goldberg Variations*
(unit of the horizontal axis = quaver).

Note that not all modulations show on the T-curve. Modulations do not lower the degree of tonality of a piece if all the pitch classes of the tonal diatonic set of the new key have sounded between the first appearance of the altered pitch class and the last time the unaltered pitch class has sounded. We could term these modulations "careful modulations". In bar 9 of the *Aria*, a C sharp is introduced (see Example 3.36), suggesting a modulation to D major. But since the last C natural (before the appearance of C sharp) sounded in bar 7 and the other six pitch classes that belong to the [7-35] sets of both G major and D major have sounded between this last C and the first C sharp (bar 8 and the first two beats of bar 9 contain, D, E, F sharp, G, A and B), there is no moment in bar 9 where not a complete tonal diatonic set can be perceived, and therefore the degree of tonality does not drop in bar 9. A similar careful modulation occurs when C natural is reintroduced at the end of bar 13, so again the degree of tonality stays unaffected, as can be seen in the T-curve in Example 3.35.

Example 3.36: Johann Sebastian Bach, *Aria* from *Goldberg Variations* BWV 988, bar 8-14.

3.5.2 Highly chromatic 'tonal' music

The T-curve in Example 3.35 is typical for highly diatonic music (music without modulations or with predominantly careful modulations). Highly chromatic tonal music on the other hand usually has a T-curve that looks rather different, as can be illustrated with the T-analysis of the 25th *Goldberg* variation. This variation starts with G and B flat on the first beat, forming ic 3 (a minor third) or an instance of set class [2-3] (see Example 3.37).

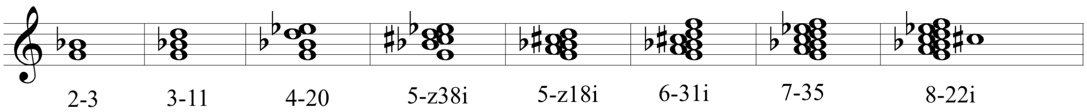
Example 3.37: Johann Sebastian Bach, *Variation 25* from *Goldberg Variations* BWV 988, bar 1-4.

The degree of tonality of ic 3 is $T_{(2\text{-}3)} = 14$. After a semiquaver a D is added turning pc-set [2-3] into a minor triad (set class [3-11]) with $T_{(3\text{-}11)} = 22$.[177] The addition of the grace note E flat raises the degree of tonality further to $T_{(4\text{-}20)} = 23$. The chromatic note C sharp lowers the degree of tonality only slightly to $T_{(5\text{-}z38i)} = 22$. The degree of tonality stays constant then until the introduction of A (demi-semiquaver on the third beat). When this A is added to the pc-set that was built up so far, we obtain [6-z43i] ($T_{(6\text{-}z43i)} = 33$). This is not the highest possible degree of tonality at that moment however: when the grace note E flat on the third semiquaver of the piece is left out we are left with a pc-set containing A, B flat

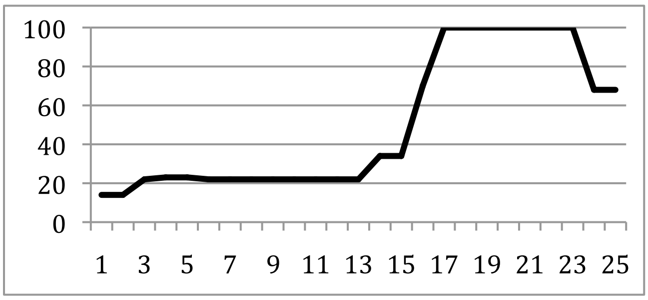
[177] "For an acculturated listener, a major or minor triad, sounded in isolation and without prior context, signals the tonic status of its root by default. In a process first described by Gottfried Weber […], a listener spontaneously imagines an isolated triad housed within a diatonic collection, signifying a tonic that bears its name." (Richard Cohn, *Audacious Euphony, Chromaticism and the Triad's Second Nature*, Oxford University Press, 2012, p. 8, reffering to Gottfried Weber, *Attempt at a Systematically Arranged Theory of Musical Composition*, Vol. 1, Translated from the 3rd edition by James F. Warner. Boston: J.H. Wilkins & R.B. Carter. Originally published as *Versuch einer geordneten Theorie der Tonsetzkunst*, B. Schott, 1846 (1817-21)).

G, D and C sharp, an instance of [5-z18i] with $T_{(5\text{-}z18i)} = 34$, which is a little bit higher; a difference that normally has theoretical importance only.[178] It the present case however there is an extra argument to leave out E flat: it was 'only' a grace note and is perceived as such by tonally acculturated listeners who are acquainted with baroque idioms. The last semiquaver F on the second beat of the first bar changes [5-z18i] into [6-31i]. At this point, he degree of tonality takes a leap to $T_{(6\text{-}31i)} = 70$. At the start of the third beat of bar 1, E flat and C are added. Leaving out C sharp results in a complete tonal diatonic set [7-35] with $T_{(7\text{-}35)} = 100$.

After a short chromatic moment at the beginning of the piece, the variation has become purely diatonic at this point, but the diatonicism is short-lived. The very last note in bar 1, C sharp is the onset of a new series of chromatic changes. It adds a pitch class to [7-35] resulting in [8-22i] with $T_{(8\text{-}22i)} = 68$. Unlike in bar 9 of the Aria, leaving out C natural is not an option to obtain a pc-set with a higher degree of tonality, because only three other pitch classes (D, B and G) have sounded since the last occurrence of C natural. Example 3.38 lists all the pc-sets occurring in the tonality analysis of the first bar of *Goldberg Variation 25*. Example 3.39 shows the corresponding T-curve.[179]



Example 3.38: pc-sets for tonality analysis of bar 1
of Johann Sebastian Bach's, *Variation 25* from *Goldberg Variations* BWV 988.



Example 3.39: T-curve for bar 1of Johann Sebastian Bach's *Variation 25*
from *Goldberg Variations* BWV 988
(unit of the horizontal axis = demi-semiquaver).

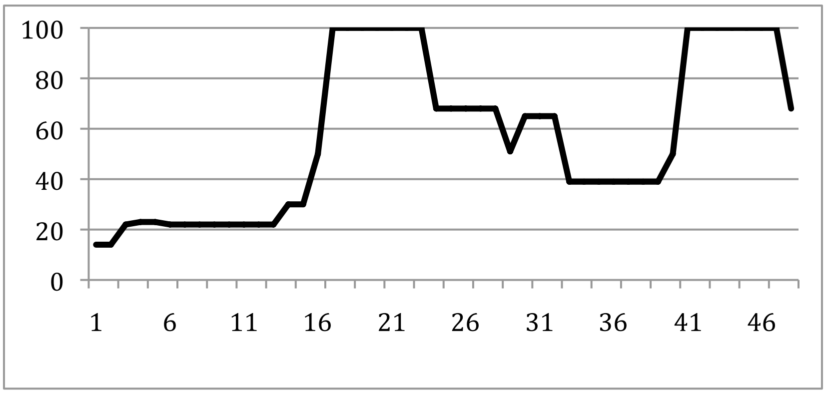
On the second quaver of bar 29, F sharp is introduced. The pitch class set with the highest degree of tonality ending on this F sharp contains F sharp, C, D, A, and G, which is an instance of [5-29] with $T_{(5\text{-}29)} = 51$. A demi-semiquaver later the addition of B natural results in pc-set containing A, G, C, D, F sharp, and B, an instance of [6-z25] with $T_{(6\text{-}z25)} = 65$. The degree of tonality is rising again. But then, after the grace note (D) on the second beat of bar 2, a very unexpected chromatic change (possibly a modulation) occurs when A flat is sounded. most listeners experience this as a surprise, and would probably expect A natural instead of A flat. This is reflected in the drop in T-value to $T_{(8\text{-}13)} = 39$. The degree of tonality starts to rise again only when E flat occurs (E flat, F, C, G, A flat) with $T_{(5\text{-}27i)} = 50$. On the last beat of bar 2, the whole 7-set [7-35] is reached again (D flat, F, B flat, E flat,

[178] In practice, the difference in degree of tonality between $T_{(6\text{-}z43i)} = 33$ and $T_{(5\text{-}z18i)} = 34$ is unperceivable. Furthermore, a slightly different T-formula may even have changed the relative order of degrees of tonality of two sets if the sets are tonally so similar.

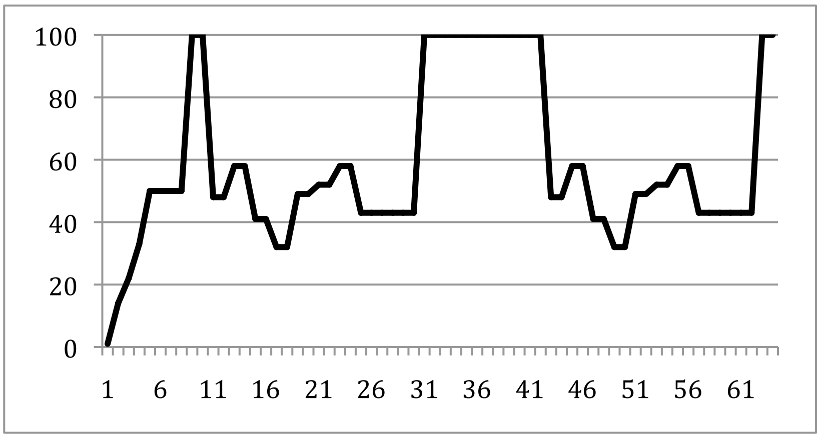
[179] The scale of the curve in Example 3.39 may suggest that it takes a long time before the highest degree of tonality is reached, but this is only seemingly so. Even though the chromatic note C sharp keeps the degree of tonality down, the maximum degree of tonality is already reached after two beats into the piece.

C, G, A flat) and the degree of tonality becomes $T_{(7\text{-}35)} = 100$, only to drop again to $T_{(8\text{-}22i)} = 68$ when B natural is added on the very last demisemiquaver of the bar, exactly like at the end of bar 1.

The average degree of tonality for the first two bars of the *Variation 25* is $T_{(average)}$ = 57,48. Starting from the first time T reaches 100 the average is even higher: $T_{(average)}$ = 74,28. Although this piece is highly chromatic, its average degree of tonality is still high. The piece may therefore be considered to be highly tonal. The T-curve for the first two bars of *Variation 25* is shown in Example 3.40. It has the typical curve for highly chromatic tonal music: always building up to the highest degree of tonality, but dropping again.



Example 3.40: T-curve for bar 1 and 2 of Johann Sebastian Bach's
Variation 25 from *Goldberg Variations* BWV 988
(unit of the horizontal axis = demi-semiquaver).



Example 3.41: T-curve for bar 1 and 2 of Johann Sebastian Bach's
Praeludium in a BWV 889 from *Das Wohltemperierte Klavier II*
(unit of the horizontal axis = demi-semiquaver).

Example 3.41 shows the T-curve of the first two bars of another highly chromatic tonal piece, Bach's *Praeludium in a* BWV 889 from *Das Wohltemperierte Klavier book I*. Many more examples of this approach can be found in the œuvre of Bach. Arnold Schoenberg used the theme of the *Fugue in b minor* BWV 869 from *Das Wohltemperierte Klavier book 1,* and several others as examples to support his claim that Bach was the first twelve-tone composer, because of the highly chromatic nature of his music.[180] Schoenberg also claimed that "in the works of earlier composers many passages of extended

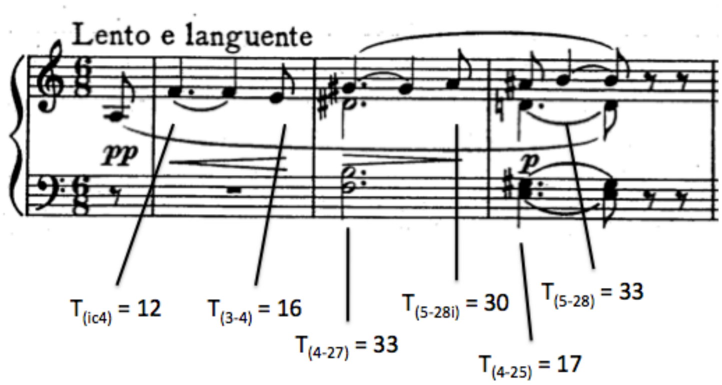
[180] See: Arnold Schoenberg, *Style and Idea*, edited by Leonard Stein, Translated by Leo Black, University of California Press, 1975, p. 393.

tonality are to be found."[181] Extended tonality is the analogue of lowered degree of tonality. There is no doubt, however, that these works should be called 'tonal' in the common sense of the term. Doubt about the tonal nature of music started to occur during the course of the nineteenth century in the tonally ambiguous chromaticism in the music of Richard Wagner, Franz Liszt and others.[182]

3.5.3 Extended tonality

The opening bars of the prelude to Richard Wagner's opera *Tristan and Isolde* (1859) constitute "perhaps the most famous, surely the most commented-on, single phrase ever written"[183]. The harmonic ambiguity of these bars raises questions about whether this music should be called tonal or not. Indeed, there is hardly ever a confirmation of a single key caused by "unresolved Dominantness"[184], the highly chromatic nature of the melodic lines make it at times hard to unambiguously determine which notes belong to the key the piece is in at a certain moment and which ones are 'chromatic'. The ambiguity of Wagner's opera is illustrated most clearly in the Tristan-chord (as the chord F-B-D sharp-G sharp in bar 2 is traditionally called (see Example 3.42), which has been interpreted in many different ways; some theorists (Dominique Jameux, for instance) even claim there is no such thing as a Tristan chord, as it is "the mere result of counterpoint by contrary movement".[185]

A T-analysis of the opening bars shows that there is a gradual rise in the degree of tonality, but it never reaches the maximum before long. In the first phrase, the Tristan-chord, which is an instance of [4-27], brings the degree of tonality to a modest $T_{(4\text{-}27)} = 33$, after which it descends back to $T_{(4\text{-}25)} = 17$ at the beginning of the next bar, where new harmonic ambiguity is created, only to regain $T_{(5\text{-}28)} = 33$ when A sharp resolves in B and is added to the [4-25] set at the beginning of the bar (see Example 3.42).



Example 3.42: T-values for the opening bars (1-3) of Wagner's *Tristan and Isolde*.

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[181] Arnold Schoenberg, *Structural Function of Harmony*, W.W. Norton & Company, revised edition 1969, p. 79.

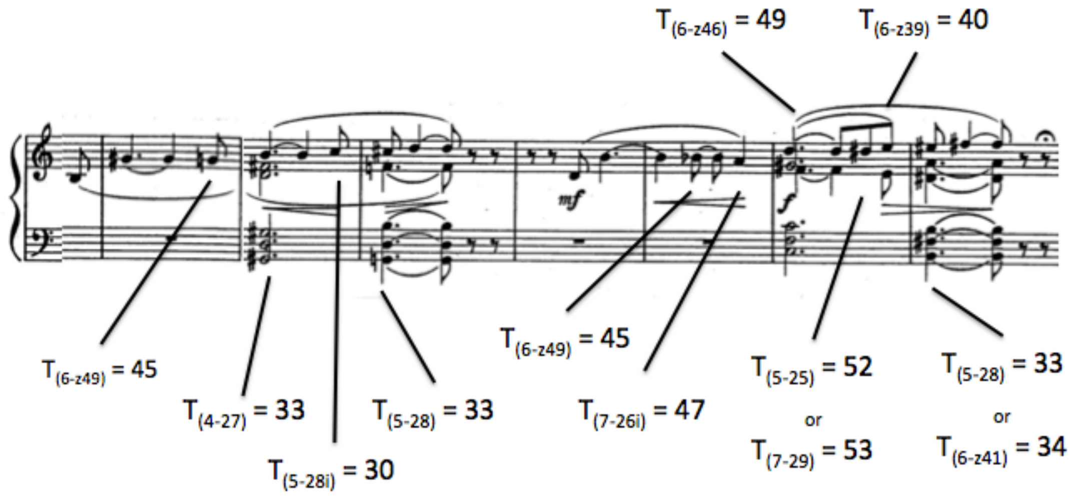
[182] "Neo-Riemannian theory [as elaborated in the writings of a.o. David Lewin and Richard Cohn] arose in response to analytical problems posed by chromatic music that is triadic but not altogether tonally unified. Such characteristics are primarily identified with music of Wagner, Liszt, and the subsequent generations, but are also represented by some passages from Mozart, Schubert, and other pre-1850 composers" (Richard Cohn, *Introduction to Neo-Riemannian Theory: A Survey and a Historical Perspective*, in Journal of Music Theory, Vol. 42, N°. 2, Neo-Riemannian Theory, 1998, pp. 167-8), Whereas neo-Riemannian theory mainly addresses triadic music ("chromatic tonality"), stressing harmonic transformations and the relations between harmonies independent of a tonic, the T-formula applies to a broader range of music, but is more limited in its analytic scope. It has a completely different goal altogether. (see also Steven Rings, *Tonality and Transformation*, Oxford University Press, 2011, and Richard Cohn, *Audacious Euphony, Chromaticism and the Consonant Triad's Second Nature*, Oxford University Press, 2012).

[183] Richard Taruskin, *The Oxford History of Western Music, Volume 3: Music in the Nineteenth Century,* Oxford University Press, 2010, p. 540.

[184] Daniel Harrison, *Harmonic Function in Chromatic Music, A Renewed Dualist Theory and an Account of its Precedents*, The University of Chicago Press, 1994, p. 155.

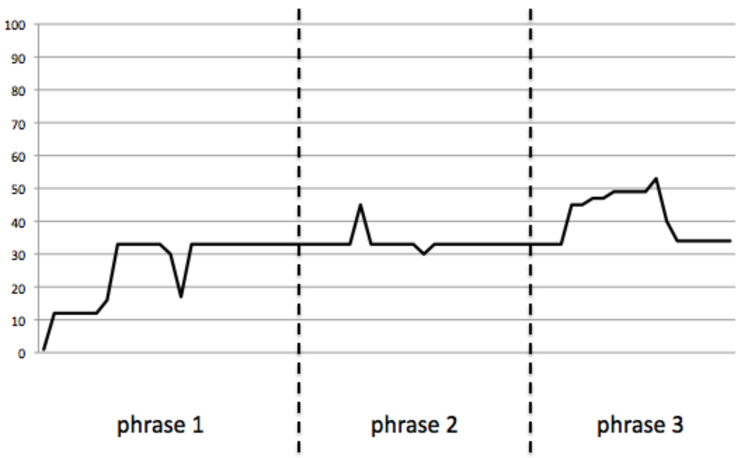
[185] See: Claude Abromont & Eugène de Montalembert, *Guide de la Théorie de la Musique*, Fayard/Henry Lemoine, 2001, p. 310.

The next phrase (upbeat to bar 5) starts with a repeat of the pitch class (B) that ended the first phrase. Therefore the T-value stays unaltered. The following G sharp was also part of the [5-28] set that ended the previous phrase, so still no change occurs. The addition of G natural at the end of bar 5 turns [5-28] into [6-z49], raising the degree of tonality to its highest level so far: $T_{(6\text{-}z49)} = 45$. The new harmonic ambiguity at the beginning of the next bar lowers the degree of tonality back to $T_{(4\text{-}27)} = 33$. It stays rather constant then until the end of the phrase. In the next phrase the degree of tonality reaches a new high in bar 10 ($T_{(5\text{-}25)} = 52$ or $T_{(7\text{-}29)} = 53$, two very similar values) as can be seen in Example 3.43.



Example 3.43: T-values for the opening bars (5-11) of *Tristan and Isolde*.

The T-curve of the first eleven bars of the Prelude to *Tristan and Isolde* (Example 3.44) shows how in every consecutive phrase the highest degree of tonality increases (33 in phrase 1, 45 in phrase 2, and 53 in phrase 3). Therefore, although the degree of tonality never reaches the maximum of 100 (which causes the doubt whether the piece can be called tonal), an expectation is created in the perception of the listeners that the piece may become more and more tonal. This expectation is met in bar 21, where the highest degree of tonality ($T_{(7\text{-}32)} = 100$) is reached for the first time (see Example 3.45).



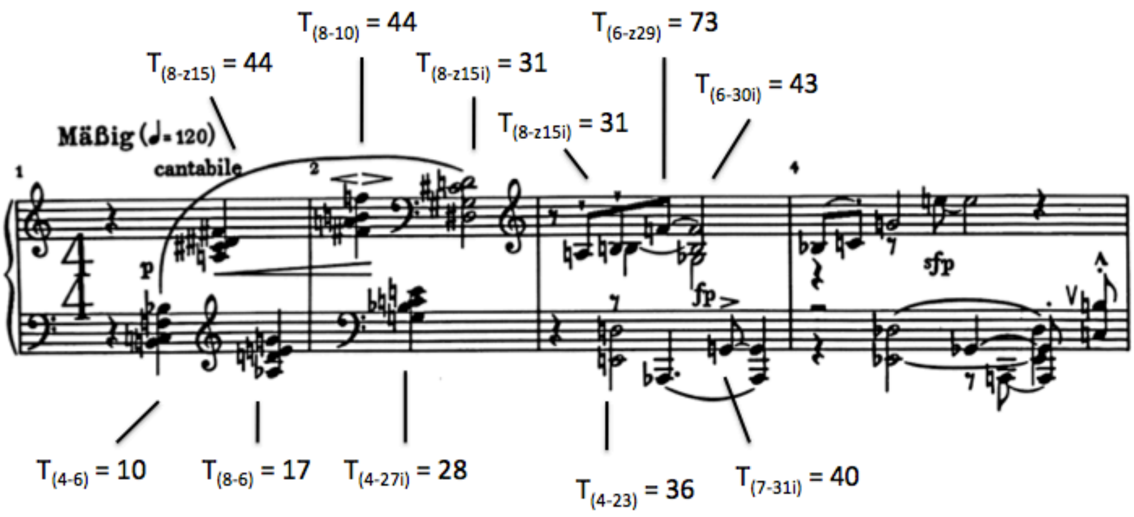
Example 3.44: T-curve for bar 1-11 of Wagner's *Tristan and Isolde*.

Example 3.45: Bar 18-21 of *Tristan and Isolde*.
The degree of tonality reaches its maximum ($T_{(7\text{-}32)}$ = 100) for the first time in bar 21.

Similar tendencies can be observed in other tonally 'problematic' music from Franz Liszt (for instance in his *Bagatelle sans tonalité* S216a from 1885) to Max Reger (Reger's *Cello sonata N°3 in F major* op. 78 from 1904, for instance reaches the maximum degree of tonality after highly complex divagations and digressions). A typical T-curve of the highly chromatic late nineteenth century music of what is sometimes called 'extended tonality' tends towards the highest degree of tonality but only rarely completely reaches it, only to abandon it immediately. It is only when the tendency towards maximum degree of tonality disappears and the T-value never reaches 100 (or only on very rare occasions) that the threshold of atonality is reached or crossed. This is the case in much of Schoenberg's pre-dodecaphonic music from after 1908 (the year of composition of his Second String Quartet). The next analysis illustrates this.

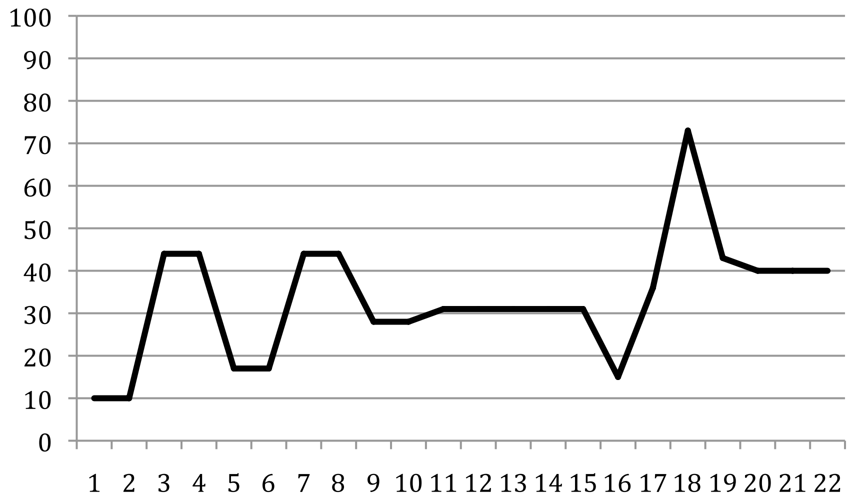
3.5.4 'Atonal' music

Arnold Schoenberg's *Klavierstück* op. 33a (composed in 1928-1929) is a dodecaphonic composition. Unlike Alban Berg's approach to dodecaphony, Schoenberg avoids links with tonality in his twelve-tone pieces. Indeed, the degree of tonality of his music stays low.



Example 3.46: T-values for Arnold Schoenberg, *Klavierstück* op. 33a (bar 1-4).

Example 3.46 shows the first four bars of the score of op. 33a with indication of the T-values for the first three bars. The T-curve corresponding to those three bars is given in Example 3.47. Note that, however the degree of tonality fluctuates heavily, there is no tendency towards T=100, as was the case in highly chromatic 'tonal' music. Peaks like the one in bar 3 of $T_{(6\text{-}z29)} = 73$ do occur, but on average the degree of tonality stays continuously low. The average degree of tonality for the first three bars is 33,01, and there is no tendency towards higher degrees of tonality in the rest of the piece. This is why Schoenberg's *Klavierstück* op. 33a and similar pieces are commonly called 'atonal', meaning: 'having a low degree of tonality throughout'.



Example 3.47: T-curve of Schoenberg's *Klavierstück* op. 33a, bar 1-3
(unit of the horizontal axis = quaver).

It is interesting to note that Paul Hindemith makes a harmonic analysis of bars 19 to 29 of Schoenberg's op. 33a.[186] Although this approach may seem a little farfetched in the case of Schoenberg's music, it shows that, even in highly atonal music, it is apparently possible for tonally 'biased' listeners to perceive tonal functions.

3.5.5 highly atonal music

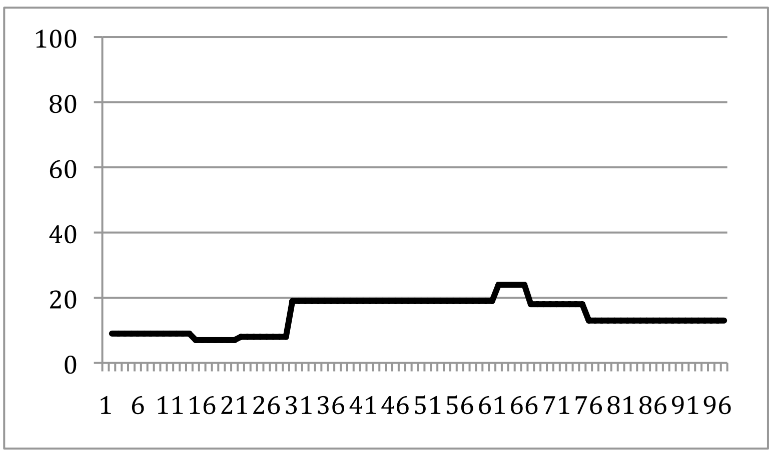
If it is true that CIG-serialism yields highly atonal music, as is the central claim of the present dissertation, this should be reflected in the T-curve of pieces written with that technique. A look at the T-curve of the first seven bars of my piece *Après la pluie*, for piano and live electronics (2008) should provide clarification. As can be seen in Example 3.48, the piece starts with pitch classes E and D sharp, interval class 1 with degree of tonality $T_{(2\text{-}1)} = 9$. The whole first bar of this piece is based on the same instance of [2-1]. Therefore the degree of tonality stays constant during this whole bar. When F is introduced during the first beat of the second bar, the degree of tonality, which was already low, decreases to $T_{(3\text{-}1)} = 7$. This is slightly lower than $T_{(2\text{-}1)} = 9$, therefore whenever only two notes sound together only pc-set [2-1] could be taken into consideration. However, since the pedal is extensively used, it seems more appropriate to consider [3-1]. On the third beat of bar 2 the degree of tonality moves to $T_{(4\text{-}1)} = 8$. When A flat is introduced in bar 3, the pc-set with the highest degree of tonality at that point is again an instance of [4-1], so the degree of tonality stays $T_{(4\text{-}1)} = 8$ until the occurrence of C sharp on the first beat of bar 6. The pc-set with the highest degree of tonality at this point is—maybe surprisingly—a hexachord: [6-z36i], consisting of (C sharp, G, A flat,

[186] Paul Hindemith, *The Craft of Musical Composition, Book 1: Theory,* Schott & Co., 1942, pp. 217-9.

F sharp, F and E), with $T_{(6\text{-}z36i)} = 24$. This sudden rise in degree of tonality occurs as a result of a single 'non-chromatic' note (C sharp). Indeed, until then every new note added to the extension of a chromatic cluster (E, D sharp, F, G flat, G, A flat); C sharp creates a gap in this cluster, which is reflected in an (albeit limited) increase in degree of tonality. The introduction of D in the middle of bar 6 lowers the degree of tonality back to $T_{(6\text{-}5i)} = 18$, and finally, in bar 7, it becomes a modest $T_{(7\text{-}5i)} = 13$.

Example 3.48: T-values for *Après la Pluie,* bar 1-7.

The T-curve for the first seven bars of *Après la pluie* is shown in Example 3.49. Characteristic for this T-curve (and for the T-curves of all my works written with the technique of CIG-serialism) are the slow fluctuations of degree of tonality. These are a result of the fact that my idiom is based on the use of central notes and CIG's (the series notes and the notes surrounding them) over a relatively long period of time. T-curves of my pieces characteristically tent to contain a lot of 'plateaus', where the degree of tonality stays constant. This contrasts strongly with the T-curves of Schoenberg's works, where pitch classes and pitch class sets change much faster (as can be seen in Example 3.46), resulting in strong fluctuations as in Example 3.47.



Example 3.49: T-curve for *Après la Pluie*, for piano and live electronics, bar 1-7.

More important than the stability of T-values is the fact that the degree of tonality stays very low in *Après la Pluie*. Even with the short 'outburst' of $T_{(6\text{-}z36i)} = 24$, the average degree of tonality for the first seven bars of the piece is $T_{(average)} = 14{,}65$, less than half the value of Schoenberg's op. 33a. It seems therefore appropriate to conclude that CIG-serialism does indeed yield highly atonal music.

3.6 A note on polytonality

To conclude the assessment of tonality, some remarks on polytonality (the simultaneous occurrence of two or more different keys in a piece of music) seem appropriate. According to Mark Delaere, the "predominant music-historical interpretation of polytonality [in the 1920s] is best summarized in the following quotation from Darius Milhaud"[187]:

> At this time (1910) I clearly felt the existence of two parallel traditions in the recent evolution of European music:
> 1. The Latin one, based on the affirmation of tonality, with its themes always clearly expressed in intervals belonging to major or minor scales or the two together. This tradition, in its normal course, was about to produce polytonality, wherein different keys are used simultaneously, each of them, however, retaining its purely tonal character.
> 2. The German one, which since Wagner had been based on an urge for constant change of the tonal centre, the orientation of the shifting harmonic material (and consequently the identity of each new centre) being made evident by the introduction of chords of the dominant seventh. The sequences and incessant modulation characteristic of the music of the Germans led them inevitably to the chromatic scale.[188]

The "German tradition" corresponds with the idea of extended tonality addressed above (in Section 3.5.3). It is in the sense of Milhaud's "Latin tradition" that polytonality is understood in the present context. Delaere summarizes "the criteria for polytonal writing put forward in what [he describes] as the collective pursuit of a theory of polytonality during the 1920s in France"[189] as follows:

> 1. Use diatonic pitch material and the triad (or, one may add, any other classified tonal chord) as its harmonic expression only. Classified chords should preferably be in root position and clearly separated from one another in musical space.
> 2. Combined keys should be related as remotely as possible; compound chords should have at least two chromatically related pitches.
> 3. If rapid and unambiguous, modulations are possible.
> 4. A polytonal composition preferably begins in one key, a second layer being added only later on. It takes time to establish different tonal centres; a single compound chord will not do.
> 5. With the exception of Monier, all authors consider bitonality on perceptual grounds as the preferred form of polytonality.
> 6. Contrasting textures, rhythms, registers and instruments help to perceive tonal polarity.[190]

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[187] Mark Delaere, *'Autant de compositeurs, autant de polytonalités différentes':Polytonality in French Music Theory and Composition of the 1920s*, in Felix Wörner, Ullrich Scheideler and Philip Rupprecht (ed.), Tonality 1900-1950. Concept and Practice, Stuttgart: Franz Steiner Verlag, 2012, p. 157.

[188] Darius Milhaud, *To Arnold Schoenberg on his Seventieth Birthday. Personal Recollections*, in : *The Musical Quarterly* XXX/4, 1944, p. 380, quoted in Mark Delaere, *'Autant de compositeurs, autant de polytonalités différentes'*, pp. 157-8.

[189] Mark Delaere, *'Autant de compositeurs, autant de polytonalités différentes'*, p. 163.

[190] Mark Delaere, *'Autant de compositeurs, autant de polytonalités différentes'*, p. 163.

It is remarkable that both Milhaud and Delaere define polytonality in terms of the use of "intervals belonging to major and minor scales or the two together", and of "diatonic pitch material". Delaere specifies further that "the exclusive use of diatonic pitch material in each tonal layer and of the triad as the basic unit of harmonic expression within those tonal layers" is "common to all polytonal music"[191], and that according to Georges Monier, a composer and music editor who "had been influential in establishing modern music in Belgium during the early 1920s [...] [c]hromaticism is strictly forbidden, since it endangers tonal identification."[192]

To assess Delaere's sixth criterion, I have composed three fragments for research purpose called *Polytonal variations* (see Appendix 5). The fragments are written for flute, viola da gamba and piano. In the first variation, the three instruments play in clearly distinct meters, 'keys' and rhythms: the flute plays long note values in slow legato quadruple time based on the pitch classes of the complete tonal diatonic set of C Major; the Gamba plays a simple five beat rhythmic ostinato (using the pitch classes of the complete tonal diatonic set of E Major) and the piano plays a hectic and irregular staccato rhythm with no perceivable meter (based on the complete tonal diatonic set of A flat Major). The registers of the three instrumental lines never overlap. It appeared to be quite easy to discern the three tonal layers and perceive them as being highly tonal.[193] In the second variation, two out of the three parts are transposed in order to merge their registers. The perceptibility of polytonality proved to be harder than in the first variation. The third variation is based on exactly the same material as the second, but this material is distributes randomly over the three instruments. The perception of polytonality becomes problematic at this stage, since the layers no longer have contrasting textures.

The approach of the TA-technique described above does not take into consideration the possibility of polytonality. It results in T-values for polytonal music that are in most cases[194] too low according to the actual perception by the listeners. Indeed, when Delaere's criteria 2 to 6 are present in a polytonal piece (as is the case in the first of the *Polytonal Variations*), the listeners will be able to distinguish the different tonal layers and perceive them as such. They will not perceive the piece as atonal, although T-analysis may result in very low T-values. Therefore, whenever music can be perceived as polytonal, T-analysis should be performed on the different tonal layers independently.

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[191] Mark Delaere, *'Autant de compositeurs, autant de polytonalités différentes'*, p. 159.
[192] Mark Delaere, *'Autant de compositeurs, autant de polytonalités différentes',* p. 162.
[193] The first and third variation were performed during an experiment by Hans Roels at the Hogeschool Gent as part of his research on "hyper-polyphony".
[194] This is certainly the case when the simultaneous keys are harmonically far apart, having few different pitch classes in common, such as when the set of C Major and F sharp Major are used at the same time.

Chapter 4. Consonance and dissonance

4.1 Introduction

The second assumption in the development of CIG-serialism as a means to obtain an atonal and dissonant musical idiom was the assertion that ic 1 is the most dissonant interval class. The claim that an interval class can be *the most* dissonant suggests that dissonance (or consonance as the opposite of dissonance) can be quantified in some way, as is the case with tonality. The first objective of the present chapter is to develop a method to quantify consonance. In order to do this, the concept of consonance has to be defined in a way that allows for such quantification whilst still corresponding with generally accepted definitions of consonance. Only after clear definition can the development of a formula for the quantification of consonance be attempted. This formula can then be implemented to assess the claim that CIG-serialism results in a highly dissonant sound idiom.

The idea of quantification of consonance is not as controversial as is the idea of quantifying tonality. As will be discussed, several researchers have attempted to elaborate such quantification, albeit with different but mostly similar outcomes.

4.2 Definitions and types of consonance

As was the case for the concept of tonality, the concept of consonance has a long history and is highly tradition-laden; therefore many definitions have been developed over the centuries. Today, the term ‘consonance’ is used with several different meanings. Still, a thorough assessment of the definitions of consonance reveals that the concept is less controversial than the concept of tonality, although—as will be seen—the criteria for defining consonance may differ. This is because the different meanings of the term ‘consonance’ can, by and large, be clearly distinguished by adding specifying adjectives. Thus, a clear distinction can be made between harmonic and melodic consonance or between musical and sensory consonance. Before determining the exact meaning of the concept of consonance used in the present context, a concise overview of different types of consonance is discussed next.

4.2.1 Harmonic and melodic consonance

The term ‘consonance’ presupposes the presence of at least two tones—one tone sounding in combination with (‘*con-sonare*’) at least one other tone.[195] The two (or more) tones may sound simultaneously or consecutively. When two tones sound simultaneously they are said to form a harmonic interval; when they sound consecutively they form a melodic interval. The consonance of the intervals is called harmonic consonance in the former case, melodic consonance in the latter.

Melodic consonance is a feature of a melodic interval. Whether a melodic interval is perceived as (or called) consonant or dissonant depends on the idiomatic context the interval occurs in, and is generally related to common-practice tonality. In the idioms of common-practice tonality all augmented and diminished melodic intervals (including the tritone) are considered dissonant. This is not the case in

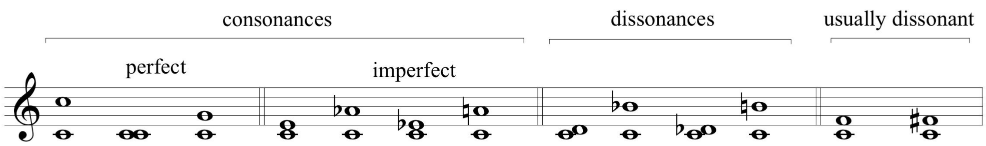
[195] As will be discussed, in tonal harmony, single notes are sometimes called dissonants when they require resolution and in many cases preparation, but even this dissonance has to be interpreted as ‘dissonance within a chord or context’. A seventh, for instance, is a dissonant in a seventh chord because the seventh and the fundamental note of the chord form a dissonant interval.

atonal music, because in atonal music "notes that are harmonically equivalent (like B flat and A sharp) are also functionally equivalent"[196]. This feature is called **enharmonic equivalence**. It entails that there are no augmented or diminished intervals, nor are there any major, minor or perfect intervals. All intervals in atonal music only have an interval content of a particular amount of semi-tones (indicated by the pitch or pitch class interval number). Intervals are determined by their content, not by interval names, and by pitch class numbers, not by note names. E sharp, F natural and G double flat are instances of the same pitch class. One might as well claim that all intervals can be at the same time a selection of major, minor, perfect, augmented or diminished, and even double or triple diminished or augmented, as can be seen in Example 4.1 below, showing different representations of the same pitch interval.[197] Since all representations in Example 4.1 are equivalent, it is impossible to decide whether the interval—here represented as a harmonic interval, but the same applies to melodic intervals—is a minor third, an augmented second, a double diminished fourth, or a triple augmented unison.

Example 4.1: Representations of a pitch interval of three semitones that are equivalent in atonal music.

Harmonic consonance (or dissonance) is a feature of at least two simultaneously sounding tones, in other words, of harmonic intervals or compounds (chords). Since, in the present dissertation, consonance and dissonance are considered within a context of atonality, harmonic consonance is the only type of consonance that will be considered henceforth.

> In the common-practice period of Western art Music, we divide [harmonic] intervals into consonant intervals, either perfect or imperfect, and dissonant intervals.
> 1. Perfect intervals include the perfect unison, the perfect octave and the perfect 5th.
> 2. Imperfect consonances include 3rds and 6ths, both major and minor.
> 3. Dissonant intervals include 2nds and 7ths, both major and minor.
> 4. The perfect 4th and tritone are generally considered dissonant, but may be consonant in some contexts.[198]



Example 4.2: Classification of consonances and dissonances in common-practice tonality
(after: Robert Gaudlin, *Harmonic Practice in Tonal Music*, W.W. Norton & Company, 2nd edition 2004, p. 17).

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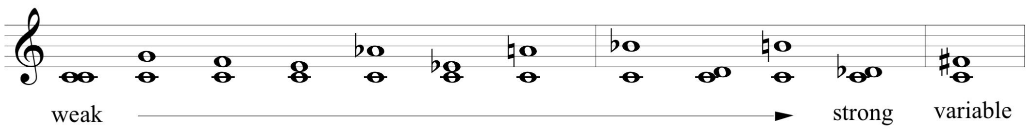
196 Joseph N. Strauss, *Introduction to Post-Tonal Theory*, Pearson Prentice Hall, 3rd edition, 2005, p. 4.

197 In practice, composers of atonal music will often use the most 'practical' representation, avoiding 'augmented' and 'diminished' representations, avoiding accidentals (certainly double sharps and flats) as much as possible, and using notations that relate to those of common-practice tonality, since most Western performers are acquainted with them. Most composers will, for instance write the melodic line B – A sharp – B, rather than B – B flat – B, since it suggests a functional relationship (possible leading note – tonic) and since it is diatonic rather than chromatic, two features that are familiar to most performers educated in the Western Classical tradition.

198 Robert Gaudlin, *Harmonic Practice in Tonal Music*, W.W. Norton & Company, 2nd edition 2004, p. 17. This division is also used in fuxian species counterpoint (see for instance: Felix Salzer, *Structural Hearing, Tonal Coherence in Music*, Dover Publications, 1952, p. 57).

Reginald Smith Brindle uses a similar subdivision and applies it to atonality. According to him, the "group of consonances (apart from the unison) consists of only three intervals (the fifth, major third, and minor third) and their inversions (fourth, minor sixth, and major sixth). The inversions are regarded as being less consonant than the original interval. […] [T]here are five dissonant intervals, but […] one of them, the tritone, can in certain situations assume a consonant character"[199]. According to Smith Brindle there are "four unambiguously dissonant intervals. The minor seventh is not as dissonant as the major second. In turn, these dissonances are mild in comparison with the harsh dissonance of the major seventh, and the still harsher minor second. Again, it will be noticed that these dissonances really comprise only two intervals (the major and minor second) and their inversions. The inversions are less dissonant than the original intervals."[200] To sum up, Reginald Smith Brindle orders harmonic intervals according to their "degree of tension" from weak to strong. The tritone has, according to him, a "variable degree of tension" (see Example 4.3).



Example 4.3: Smith Brindle's classification of intervals from weak to strong dissonants, according to their "degree of tension" (source: Reginald Smith Brindle, *Serial Composition*, Oxford University Press, 1966, p. 37).

Note the difference between the two classifications. According to Gauldin (Example 4.2), the perfect fourth is "usually" dissonant, whereas for Smith-Brindle (Example 4.3) it is a strong consonance (an interval with weak tension). Similar discrepancies will occur in the subsequent section where experimentally determined consonance indices will be discussed.

4.2.2 Musical and sensory consonance

Consonance can be further divided into musical and sensory consonance. Musical consonance is a feature of tones or tone combinations within a (musical) context; sensory consonance applies to tone combinations in isolation. Allan B. Smith claims: "It is important to recognize the differences between tonal [or sensory] consonance as an immediate, isolated property of tones pairs and key context stability [musical consonance] being dependent on the context of surrounding tones."[201]

4.2.2.1 Musical consonance

The concept of musical consonance can be interpreted in two ways. A first kind of musical consonance has to do with "tension" and "resolution". In some musical idioms—such as common-practice tonality—there are notes or chords that need to be "resolved" *in certain contexts*, which means they have to be followed by specific other notes or specific chords. All notes or note combinations that require such "resolution" in a functional context are considered dissonants. According to Carol Krumhansl, musical consonance "refers to intervals that are considered stable or free of tension, and constitute good resolution"[202]. Note that musical consonance in this sense, **resolution-based musical consonance**, may also apply to single notes (not only simultaneous combinations of notes). A leading note sounding alone, for instance, is a dissonant note in this sense.

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[199] Reginald Smith Brindle, *Serial Composition*, Oxford University Press, 1966, p. 36.
[200] Reginald Smith Brindle, *Serial Composition*, p. 36.
[201] Allan B. Smith, *A "Cumulative" Method of Quantifying Tonal Consonance in Musical Key Contexts,* in Music Perception: An Interdisciplinary Journal, Vol. 15, N°2 (Winter, 1997), University of California Press, p. 176.
[202] Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* Oxford University Press, 1990, p. 51.

Note combinations that require resolution are called “unstable”. In Roger Kamien’s wording: "An unstable tone combination is a dissonance; its tension demands an onward motion to a stable chord. Thus dissonant chords are 'active'; traditionally they have been considered harsh and have expressed pain, grief, and conflict."[203] Mieczyslaw Kolinski calls resolution-based musical consonance the “functional approach”. He states that:

> [the ‘functional’ approach] contends that an interval or chord is dissonant or consonant according to whether or not it requires a resolution. The rules for functional appreciation of consonance and dissonance have been established within the framework of traditional harmony, but even there the same interval or chord, depending on its harmonic function, may or may not require a resolution and, therefore, be dissonant in one context and consonant in another one. Even the perfect consonance of a fifth might become a pronounced ‘functional’ dissonance […].[204]

This defining of dissonance in terms of “active” chords (that need to be resolved) traditionally links consonance and dissonance to (tonal) functionality, and can therefore be called **functional consonance and dissonance.** More generally, one may claim—as Krumhansl does—that musical consonance “depends strongly on the musical style and also on the particular context in which the interval is sounded.”[205] Since my research focuses on music that is not written in a tonal idiom, functional consonance and dissonance are irrelevant within the context of the present dissertation. The definition of consonance and dissonance that is relevant within the present context should be independent of the tonal tradition.[206]

As was mentioned, the functionality-related definition(s) of musical consonance or musical dissonance can apply to single tones within a musical context. Charles Rosen claims that it is a general misconception about dissonance in music that “in order for a dissonance to exist at least two notes must be played together.”[207] This is indeed the case for musical consonance, but not for sensory consonance, as will be discussed in a moment. Rosen claims:

> It is precisely this effect of ending, this cadential function, that defines a consonance. A dissonance is a musical sound [not necessarily a simultaneous combination of sounds] that must be resolved, i.e. followed by a consonance: a consonance is a musical sound that needs no resolution, that can act as a final note, that rounds off a cadence. Which sounds are to be consonances is determined at a given historical moment by the prevailing musical style, and consonances have varied radically according to the musical system developed in each culture.[208]

Rosen further claims that dissonances “cannot be used to end a piece or even a phrase (except, of course, if one wants to make an unusual effect of something incomplete, broken off in the middle)”[209]. This depends completely on the musical idiom used, and does not apply to idioms—like mine—where there is no such thing as a musical consonance or dissonance. In such idioms, no sound ‘must’ be resolved.

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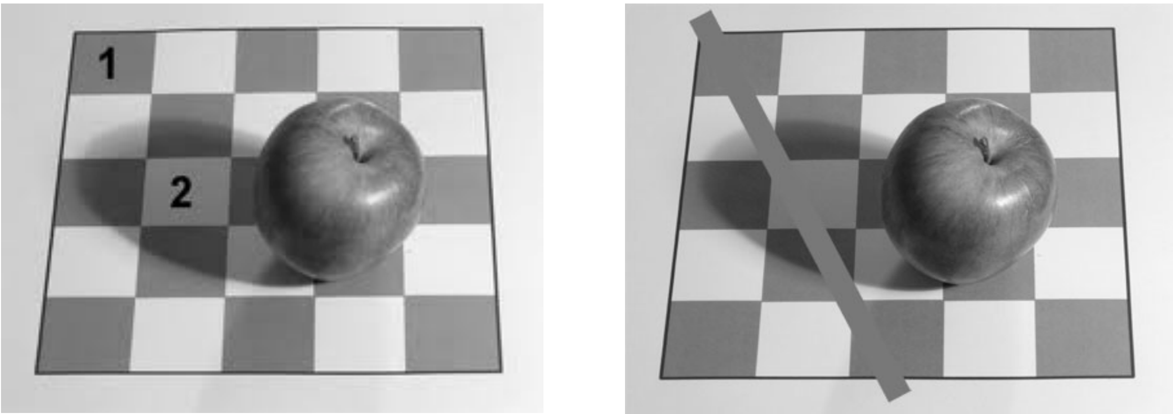
[203] Roger Kamien, *Music: An Appreciation*, Mc Graw-Hill, 6th brief Edition, 2008, p. 41.
[204] Mieczyslaw Kolinsli, *Consonance and Dissonance*, in: Ethnomusicology, Vol. 6, N°2, 1962, p. 66.
[205] Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* p. 51.
[206] Other aspects that may influence the perception of consonance and dissonance include temperament (tuning) and timbre. These aspects are not discussed here either since they are not relevant in the present context.
[207] Charles Rosen, *Arnold Schoenberg*, University of Chicago Press, 1975, p. 23.
[208] Charles Rosen, *Arnold Schoenberg*, p. 24.
[209] Charles Rosen, *Arnold Schoenberg*, p. 24.

The second type of context-related (musical) consonance and dissonance is **relative musical consonance**. This type starts from the idea of sensory consonance, but compares the 'degree' of consonance of chords or tone compounds (pc-sets) within a musical context. Whether a pc-set is perceived as consonant or dissonant depends on the consonance or dissonance of the chords preceding it.[210] Reginald Smith-Brindle writes: "After a succession of dissonances the fourth appears as a stable consonance […] but after a succession of consonant intervals, the fourth can be less stable, having a 'cadential' tendency for the upper note to resolve downwards."[211] This claim applies to any interval: after 'more' dissonant intervals, a 'more' consonant interval will appear 'very' consonant; within a highly consonant context, the same interval may be perceived as dissonant. In a piece consisting mainly of harmonic intervals of octaves and fifths (ic 5), for instance, a sudden major third (ic 4) will be perceived as dissonant compared to the intervals that precede it. However, the exact same major third will sound very consonant when it is preceded by minor second intervals (ic 1).

Relative musical consonance is comparable to the phenomenon of surrounding-influenced perception of brightness, hue and vividness in visual perception. The brightness, hue or vividness of a colour patch is perceived differently with differing surroundings.

> Vivid as the colours around us seem, their brilliance is manufactured in the eye. Our eyes gauge the brightness, hue and vividness of patches of colour by relating them to the shade, hue and vividness of their surroundings, and we can draw figures […] to show how the same colour looks very different when it appears in different surroundings.[212]

This phenomenon can be clearly illustrated by the famous example of the chess-shadow illusion shown below (Example 4.4). In Example 4.4 a, square 2 on the chessboard appears to be lighter than square 1, although both squares have the same brightness, as can be seen in Example 4.4 b. The illusion of different brightness originates from a context-based interpretation by the perceiver. Square 2 is surrounded by darker squares than square 1, making it appear brighter.



a b

Example 4.4: Chess-shadow illusion:
a) apparent different brightness of squares 1 and 2 on the chessboard;
b) the line shows that both squares have the same brightness
(source : http://www.chrismadden.co.uk/wordpress/?p=25 [last accessed: 30-10-2010]).

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[210] Chords following the chord in question can have no influence on its perceived consonance, since they have not been sounded yet.

[211] Reginald Smith Brindle, *Serial Composition*, p. 36. Smith Brindle does not clearly distinguish between musical and sensory consonance here. He uses the concept of relative musical consonance in a discussion of sensory consonance. Although relative musical consonance is related to sensory consonance, it should still be clearly distinguished from it.

[212] Simon Ings, *Matters of light and darkness*, 2009, http://simoningsmirror.wordpress.com/2009/10/26/matters-of-light-and-darkness/ [last accessed: 31 January 2014].

The chess-shadow illusion proves that the difference in brightness is not a feature of the observed object (the squares on the chessboard) but an entirely perceptual illusion. The same can be said about relative musical consonance. Whether a harmonic interval or a chord is perceived more or less consonant in this sense is not determined by the physical sound complex but by perceptual interpretation. One may therefore wonder why, in the case of visual perception, the phenomenon is called an illusion, whereas this is not done for relative musical consonance. Some auditory phenomena are recognized as illusions (e.g. binaural beats, Deutsch's scale illusion, glissando illusion, Shepard scale[213]), why not relative musical consonance?[214]

Relative musical consonance is not very relevant for the purpose of the present dissertation, since it is my aim to achieve a sound idiom with a constant high degree of dissonance. This means that the degree of dissonance of the simultaneously sounding tones is meant to be relatively constant. All harmonic intervals will therefore be perceived as more or less equally dissonant. At the end of the chapter, the concept of musical consonance will be addressed briefly again, but it will be left out of further discussion due to its restricted relevance. In the rest of the text only sensory consonance will be considered. This is discussed next.

4.2.2.2 Sensory consonance

Groves Dictionary describes sensory consonance as follows:

> 'Sensory consonance' refers to the immediate perceptual impression of a sound as being pleasant or unpleasant; it may be judged for sounds presented in isolation (without a musical context) and by people without musical training. 'Musical consonance' is related to judgments of the pleasantness or unpleasantness of sounds presented in a musical context; it depends strongly on musical experience and training, as well as on sensory consonance. These two aspects of consonance are difficult to separate, and in many situations judgments of consonance depend on an interaction of sensory processes and musical experience.
>
> Historically, some theorists have argued that the basis of perceived consonance is physiological or sensory (Helmholtz, 1863), while others have attributed it to the learning of relatively arbitrary cultural patterns (Lundin, 1947). However, one should not regard these theories as mutually exclusive. The relative importance of sensory factors and learning in a particular musical culture will depend on the types of sound being presented, on the instructions given and on the musical experience of the listeners. Psychoacoustic studies have usually emphasized sensory consonance, and tried to explain it in terms of the physical nature of the sounds and the way the sounds are analysed in the peripheral auditory system.[215]

John Fauvel claims that "two notes are *consonant* if they sound 'pleasing' when played together"[216]. David Huron and Carol Krumhansl use similar definitions. Huron defines (sensory)[217] consonance as

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[213] Personally I don't perceive the Shepard scale as a single scale, but as a continuous succession of scales.

[214] This consideration leads us to the philosophical problem of perception, more specifically to the "argument from illusion" as an argument against Direct Realism (the claim that we perceive objects as they are, that we are directly aware of "normal objects"). See for instance: A.D. Smith in *"The Problem of Perception":* "concerning the immediate object of awareness and the normal object: […] one possesses a genuine attribute that the other lacks." (A.D. Smith, *The Problem of Perception*, Harvard University Press, 2002, p. 8).

[215] Groves dictionary online, lemma "consonance", referring to Hermann von Helmholtz: *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik,* Brunswick, 1863, and: Robert W. Lundin: *Towards a Cultural Theory of Consonance*, in: Journal of Psychology, xxiii, 1947, pp. 45–9 [last accessed: 23 February 2013].

[216] John Fauvel, Raymond Flood & Robin Wilson (eds.), *Music and Mathematics, From Pythagoras to Fractals*, Oxford University Press, 2003, p. 13.

"[t]he idea that some sounds or sound-combinations are more beautiful or euphonious than others"[218], and dissonance as "[t]he idea that some sounds or sound-combinations are less euphonious than others. An ancient idea that has received numerous treatments. Psychoacoustic research supports a low-level auditory irritation dubbed 'sensory dissonance' that is related to the timbre, register, and the interval class content for sonorous moments."[219] Note that Huron refers to psychoacoustics, which suggests that interpretation is not the mere result of subjective taste. Krumhansl claims that sensory[220] consonance "refers to the attribute of particular pairs of tones that, when sounded simultaneously in isolation, produce a harmonious or pleasing effect"[221].

Linking consonance and dissonance to impressions of pleasantness or unpleasantness, beauty or euphony, may be problematic. According to Charles Rosen, it is a second general misconception about consonance that "a dissonance is a disagreeable noise"[222]. Rosen claims that dissonance is "[t]he primary means of musical expression"[223]. It is clear that he rejects the taste-based concept of sensory consonance and dissonance. I agree with this rejection, because it could turn consonance and dissonance in a merely subjective concept. I perceive ic 1 as very pleasant, but this does not make it consonant, not even for me. Mieczyslaw Kolinski states this idea as follows:

> The two most widely accepted descriptive definitions of consonance and dissonance could be termed 'aesthetic' and 'functional'. The former conceives of consonance as of a 'pleasant' or 'satisfying', and of dissonance as of a 'disagreeable' or 'shocking' simultaneous tone combination. Evidently, such an 'aesthetic' appreciation remains highly relative and subjective and largely varies according to style and individual taste, but above all the aesthetic approach does not touch the core of the problem; For example, theortists and musicians agree that a fifth is more consonant than a third though many people might prefer the sound of a third to that of a fifth.[224]

This does not mean I reject the idea of sensory consonance and dissonance altogether; I only claim that it has to be based on other criteria, independent of any aesthetic or taste-based judgment.

4.3 Criteria for sensory consonance and dissonance

Attempts to quantify the consonance of harmonic intervals have been made by several researchers, each according to their own criteria. These criteria may be subjective and assessed with experimental (mainly psychoacoustic) methods involving the judgment of other people, but they may also be objective, based on features of the sounds involved or on physiological (non-perceptual[225]) features of the auditory system.

A brief overview of some seminal experiments and calculations to determine consonance indices for intervals (values indicating the level or degree of consonance of intervals) will be given next. This is not a comprehensive overview; it is only meant to briefly sketch a context with which my own calculations can afterwards be compared.

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[217] Huron and Fauvel don't specify that their definitions apply to sensory consonance and dissonance, but it is clear from the context that this is the kind of consonance and dissonance meant.
[218] David Huron, *Sweet Anticipation: Music and the Psychology of Expectation*, MIT Press, 2006, p. 411.
[219] David Huron, *Sweet Anticipation*, p. 413.
[220] Krumhansl uses the term "tonal consonance" (see: Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* p. 51).
[221] Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* p. 51.
[222] Charles Rosen, *Arnold Schoenberg*, p. 23.
[223] Charles Rosen, *Arnold Schoenberg*, p. 23. Rosen adds that this "is true at least for Western music since the Renaissance" (p. 23).
[224] Mieczyslaw Kolinsli, *Consonance and Dissonance*, in: Ethnomusicology, Vol. 6, N°2, 1962, p. 66.
[225] Perception is here defined as "the process through which people take raw sensations from the environment and interpret them, using knowledge, experience, and understanding of the world, so that the sensations become meaningful experiences" (Douglas A. Berstein, Edward J. Roy e.a., *Psychology*, Houghton Mifflin Company, 2nd edition, 1991, p. A-26).

4.3.1 Subjective versus theoretical criteria in 'classic' experiments

Classic determination of consonance (or dissonance) indices has been done by researchers such as Hermann Helmholtz[226] (1885), Constantine Frithiof Malmberg[227] (1918), Kameoka and Kuriyagawa[228] (1969), or Hutchinson and Knopoff[229] (1979).[230] Their outcomes were synthesized by David Huron in his 1994 article *Interval-Class Content in Equally Tempered Pitch-Class sets*[231]. Karol Krumhansl gives an extensive description of the methods and criteria used in her book *Cognitive Foundations of Musical Pitch* (1990).[232] Some of those studies are theoretical, others use experimental methods based on subjective criteria for the interpretation of consonance.

Hermann Helmholtz calculated consonance indices based on assessments of "roughness… for equal-tempered and simple-ratio tunings"[233]. He promoted the hypothesis "that the difference between consonant and dissonant intervals is related to beats of adjacent partials"[234]. Reinier Plomp and Willem Levelt further developed the Helmhotz model of beats in 1965.[235]

Constantine Malmberg's studies were partly theoretical but he also carried out a series of experiments in the psychological laboratory of the State University of Iowa between 1911 and 1913. "The object of this investigation is first to establish the ranking order of the musical intervals within the octave c'c" with respect to the degree of consonance, and second, to standardize a measurement of the perception of consonance."[236] The following (subjective) criteria were used:

> For consonance:
> 1. Blending: a seeming to belong together, to agree.
> 2. Smoothness: relative freedom from beats.
> 3. Fusion: a tendency to merge into a single tone, unanalyzable.
> 4. Purity: resultant analagous [sic] to pure tone. (See Wundt.)[237]
>
> For dissonance:
> 1. Disagreement: incompatibility.
> 2. Roughness: harshness, unevenness or intermittence.
> 3. Disparateness: separateness or seeming to stand apart analyzable, "twoness".
> 4. Richness: resultant analogous to rich tone.
>
> In terms of these factors we may then define consonance as follows: When the two tones of a two-clang tend to blend or fuse and produce a relatively smooth and pure

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226 Hermann Helmholtz, *On the Sensations of Tone as a Physiological Basis for the Theory of Music,* A.J. Ellis, Ed. & Trans., Dover, 1954 (originally published in 1885).

227 Constantine Frithiof Malmberg, *The Perception of Consonance and Dissonance*, in: Psychological Monographs Vol. 25, 1918, pp. 93-133.

228 Akio Kameoka & Mamoru Kuriyagawa, *Consonance Theory Part II: Consonance of Complex Tones and its Calculation Method*, in: The Journal of the Acoustical Society of America, Vol. 45, N° 6, 1969, pp. 1460-69.

229 William Hutchinson & Leon Knopoff, *The Acoustic Component of Western Consonance,* in: Journal of New Music Research, Vol. 7, N°1, 1978, pp. 1-29.

230 Those are all discussed in Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* Oxford University Press, 1990.

231 David Huron, *Interval-Class Content in Equally Tempered Pitch-Class sets*: *Common scales exhibit optimum tonal consonance*, in *Music Perception: an Interdisciplinary Journal*, Vol. 11, N°3, 1994, pp. 289-305.

232 Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* pp. 50-62.

233 Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* p. 52.

234 Reinier Plomp & Willem J.M. Levelt, *Tonal Consonance and Critical Bandwidth*, Journal of the Acoustical Society of America, Vol. 38, 1965, p. 548.

235 Reinier Plomp & Willem J.M. Levelt, *Tonal Consonance and Critical Bandwidth*, pp. 548-60.

236 Constantine Frithiof Malmberg, *The Perception of Consonance and Dissonance*, p. 93.

237 "Restfulness a feeling of completeness, finality or satisfaction, with its opposite disquietude a feeling of incompleteness, needing to be resolved, was first adopted as a fifth criterion, but it soon developed that it must be dropped as it is a variable criterion directly due to progression and association, which must be excluded." (Constantine Frithiof Malmberg, *The Perception of Consonance and Dissonance*, p. 108).

resultant, they are said to be consonant. Dissonance is the reciprocal of this.[238]

In Malmberg's studies "[a] number of different instrumental timbres were employed […], and the criteria of consonance varied (pleasantness, smoothness, fusion, purity, rhythmic coincidences, and some purely mathematical and theoretical criteria). Despite these differences, the obtained rankings exhibited considerable convergence"[239]. One of his experiments "presented musically experienced listeners with all possible pairs of two-tone intervals (spanning an octave range or less) and recorded for each pair which interval was preferred"[240]. Malmberg's studies resulted in two tables of consonance values: a table of "ranks" and a table of "data".[241]

In Kameoka & Kuriyagawa's experimental studies, "[s]ubjects seated in an auditorium judged the difference of consonance between the paired tones and assigned one of the numbers -2, -1, 0,+1, and +2 according to the *subjective* distance in consonance"[242].

Hutchinson & Knopoff, finally, developed:

> [a] formalism […] for providing a measure of dissonance in a superposition of complex tones. The formalism is based on an extension of the Helmholtz- Plomp and Levelt model of beating as the cause of dissonance.
> For dyads this measure of dissonance gives a good fit to psychological rank orderings of dissonance and its absence (consonance), and to orderings of consonance and dissonance found in Western common practice and pedagogy. A logarithmic scale for the perception of consonance and dissonance is indicated.[243]

Example 4.5 below lists the consonance values (or indices) for all interval classes[244] obtained by the researchers discussed. Examples 4.6 a through g show a graphical representation of the values for interval class 1 to 6 (on horizontal axis). Note that the Malmberg data and Huron values are actually dissonance indices (the value rises with increasing degree of dissonance). The important element here is not so much the scale and absolute values of the consonance indices (vertical axis in the graphs), but their relative values. Those become clear in the graphs.

Interval class	Helmholtz ET	Helmholtz simple-ratio	Malmberg Ranks	Malmberg Data	Hutchinson & Knopoff	Kameoka & Kuriyagawa	Huron
1	76	70	11,29	0	0,4886	285	-1,428
2	25	32	9,5	1,5	0,269	275	-0,582
3	24	20	6,6	4,35	0,1109	255	0,594
4	18	8	4,65	6,85	0,0551	250	0,386
5	3	2	3,1	7	0,0451	245	1,24
6	18	20	8,28	3,85	0,093	265	-0,453

Example 4.5: Consonance values for six classic studies and synthesis by David Huron
(sources: Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* p. 57,
and David Huron, *Interval-Class Content in Equally Tempered Pitch-Class sets*, p. 294).

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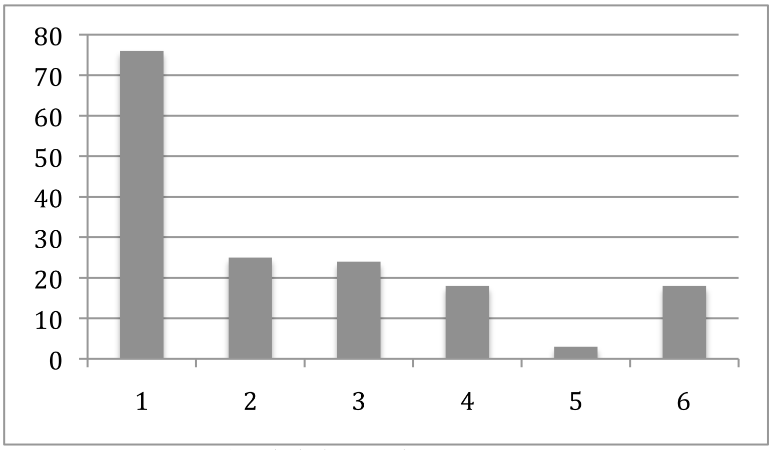
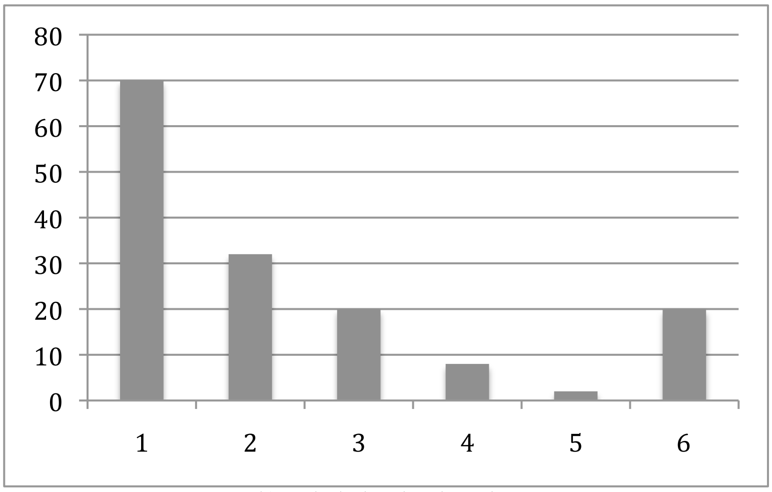
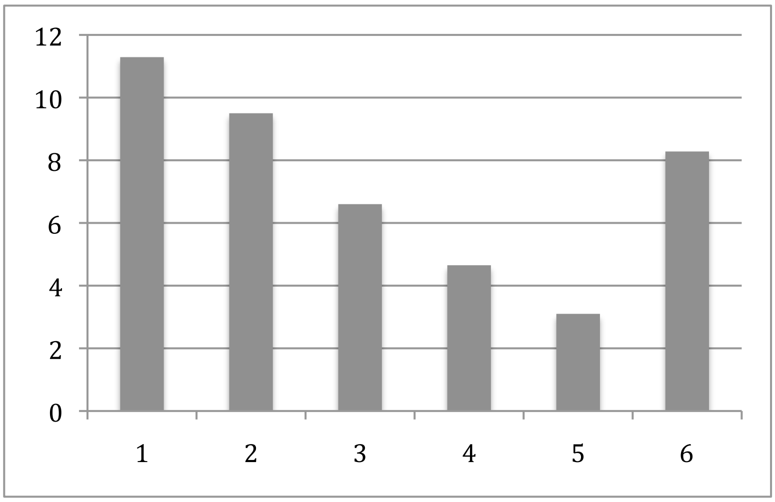
238 Constantine Frithiof Malmberg, *The Perception of Consonance and Dissonance*, p. 108.
239 Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* Oxford University Press, 1990, p. 56, referring to the table of consonances in Example 4.5 based on the results of this experiment.
240 Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* p. 56.
241 See: Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* p. 57.
242 Akio Kameoka & Mamoru Kuriyagawa, *Consonance Theory Part II: Consonance of Complex Tones and its Calculation Method*, in: The Journal of the Acoustical Society of America, Vol. 45, N° 6, 1969, p. 1464 [my italics].
243 William Hutchinson & Leon Knopoff, *The Acoustic Component of Western Consonance,* in: Journal of New Music Research, 7:1, 1978, p. 1.
244 The researchers (with the exception of David Huron) determined values for intervals rather than interval classes, but for the purpose of the present research, I restrict the results to interval classes.



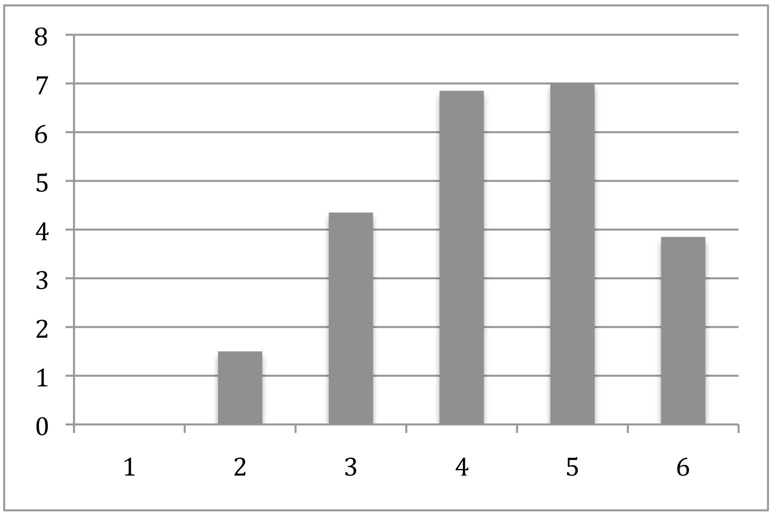
a) Helmholtz Equal Temperament.



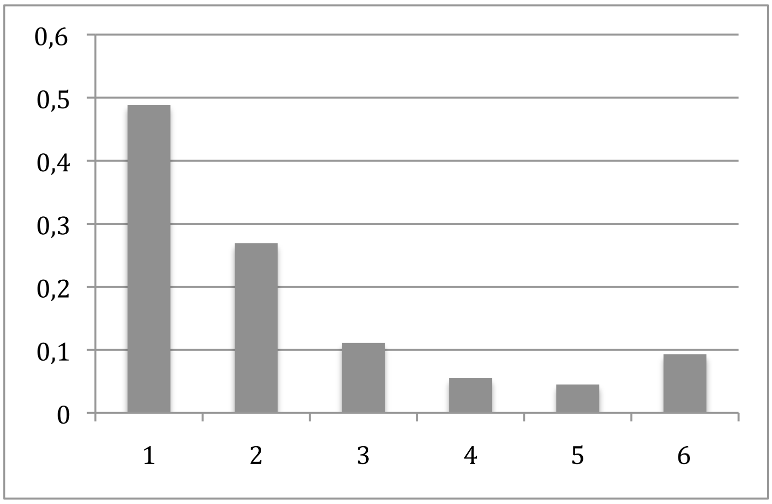
b) Helmholtz simple-ratio.



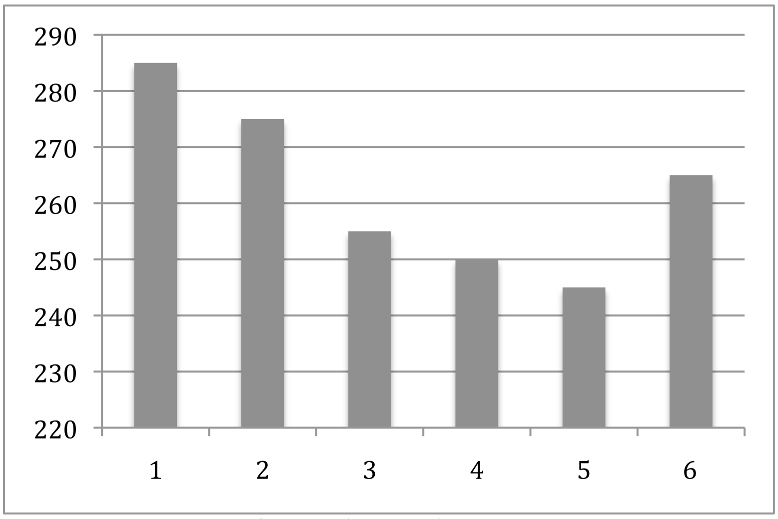
c) Malmberg ranks.



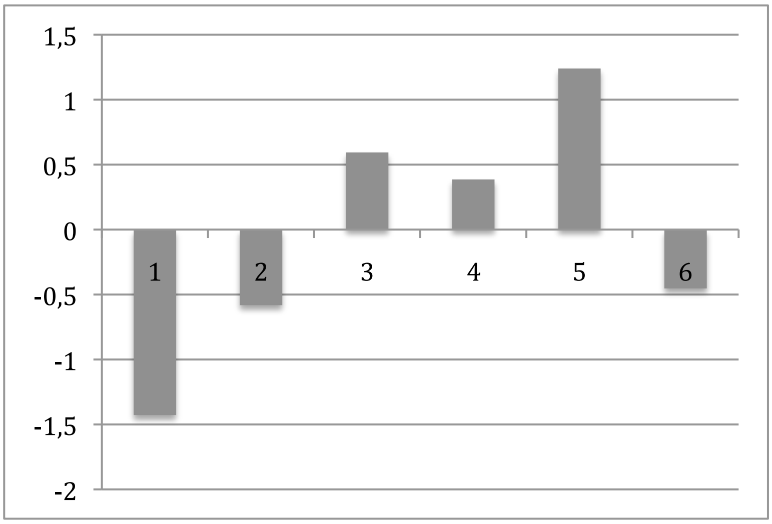
d) Malmberg data.



e) Hutchinson & Knopoff.



f) Kameoka & Kuriyagawa.



g) Huron.

Example 4.6 a-g: Consonance indices values (vertical axis) of interval classes 1 to 6 (horizontal axis) for six classic studies and synthesis by David Huron.

Comparing the results in Examples 4.6 a to g shows "substantial agreement"[245] between at least six of the seven graphs. In all graphs but Huron's (Example 4.6 h), the values decrease (they increase for Malmberg's data (Example 4.6 d)) from ic 1 to ic 5, making ic 1 the most dissonant interval class and ic 5 the most consonant. The rate of decent may differ (compare the almost linear descent in Malmberg ranks (Example 4.6 c) or the gap between ic 1 and ic 2 in Helmholtz's ET results (Example 4.6 a) with the 'logarithmic' curve of the other graphs), but the tendency is identical, with only one exception: in Huron's result ic 3 is more consonant than ic 4 (see Example 4.6 g).

As important as the similarity of values for ic 1 through ic 5 is the discrepancy for the value of the ic 6 index. Malmberg, Huron, and Kameoka & Kuriyagawa place ic 6 between ic 2 and ic 3; Helmholtz has identical values for ic 4 and ic 6 in Equal Temperament and for ic 3 and ic 6 in simple ratio; Hutchinson & Knopoff, finally, end up with a value for ic 6 between that of ic 3 and ic 4. There is, in other words, no consensus about the degree of consonance of ic 6 and its position in the ranking list. This conclusion will be important in my own calculation of consonance indices.

4.3.2 Physiological criteria and evolution

Subjective criteria of perception and judgment of consonance such as those used by some of the researchers discussed in the previous section may be partly physiological—determined exclusively by the physical characteristics of the human auditory system in the ear and the central nervous system—but they are not exclusively so—or at least, it is probably impossible to differentiate. There are, however, criteria that are purely physiological.

For instance, "Helmhotz (1885/1954) noted that, when two notes that are close but not identical in frequency are sound together, the listener has a sensation of beating or roughness."[246] This is a physiological phenomenon; it is not a result of aesthetic judgment, but of the way our auditory system functions. The idea results in the fact that consonance is register-related and "a given interval will be less consonant in lower registers."[247]

[245] Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* p. 59. Krumhansl only considers the six 'classic' studies.
[246] Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* p. 52.
[247] Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* p. 53.

This idea was later replaced by "a more general concept of interference between component frequencies within what is known as the critical bandwidth"[248]. According to Rainier Plomp and other researchers at the Dutch Institute for Perception Research, Helmholtz's idea of beating caused by tones with frequencies that lie close together was too simple.

> Slow beats do not give a sense of dissonance, but merely a rising and falling of amplitude. Further, as we gradually separate the frequencies of two sine waves or "pure tones", we hear a disagreeable roughness even when the frequencies are so far apart that we no longer distinguish beats. The range of frequencies in which we hear beats or roughness is called the *critical bandwidth*.[249]

> Greenwood (1961), and Plomp and Levelt (1965) explained the phenomenon of sensory dissonance in a somewhat different way [than Helmhotz]. Using the concept of "critical bandwidth," sensory dissonance was described as a function of the frequencies of the two pure tones, rather than as a function of the number of beats perceived. In that study […] the term "tonal consonance" is used to describe the lack of sensory dissonance. That is, a tone combination that is low in sensory dissonance is said to be high in tonal consonance.[250]

Such physiological criteria may be used to determine consonance indices. They may be termed 'objective' since no subjective (aesthetic) judgment is involved. Only the possible divergences in the way the auditory systems of separate individuals functions may account for a difference in perception, but these divergences are mostly small (at least for individuals with 'normal' auditory systems) and the differences in perception are therefore probably irrelevant.

Not only are physiological criteria highly identical for all human listeners, it is also safe to say that over the 400 years of evolution of tonality, it is not the physiologically perceived degrees of dissonance or the consonance of intervals that have evolved. 400 years are probably too little to account for the gradual evolution of the physical perception of consonance in the ear or the functioning of the auditory cortex might have undergone (especially since it is not clear which evolutionary advantage such an evolution may have had over this extremely short period of human evolution).[251] Therefore it is not unreasonable to claim that the physical (physiological) perception of consonance has not changed enough over the last 400 years to say that degrees of dissonance or the consonance indices of intervals have evolved noticeably. What has evolved is the *cultural acceptability* of consonance.

Indeed, Western music has continuously evolved towards an increasing degree of atonality and dissonance even long before the start of the twentieth century. The tendency can be observed in the evolution of consonance and 'tonality'[252] from ancient Greek times, over the Middle Ages, Renaissance, all the way to Debussy, Ravel, and Messiaen, as can be seen in the diagram in Example 4.7.

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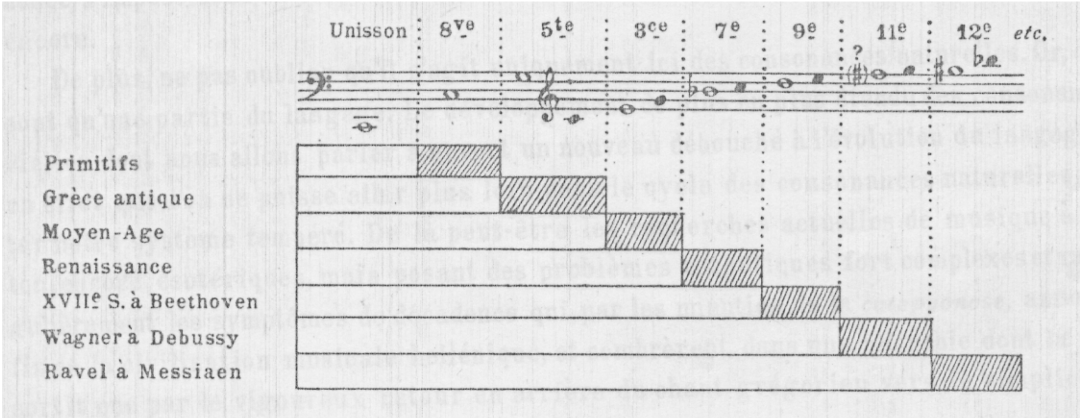
[248] Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* p. 53.

[249] John R. Pierce, *The Science of Musical Sound*, W. H. Freeman and Company, New York, 1992, pp. 78-9 [original italics].

[250] Allan B. Smith, *A "Cumulative" Method of Quantifying Tonal Consonance in Musical Key Contexts"* in Music Perception: An Interdisciplinary Journal, Vol. 15, N°2 (Winter, 1997), University of California Press., pp. 175., referring to Donald D. Greenwood, *Critical Bandwidth and the Frequency coordinates of the basilar membrane*, Journal of the Acoustical Society of America, Vol. 33, N°4), pp. 1344-56, and Reinier Plomp & Willem J.M. Levelt, *Tonal Consonance and Critical Bandwidth*, Journal of the acoustical Society of America, Vol. 38, pp. 548-60.

[251] In the case of gustatory perception (our sense of taste), it is the case that "[t]he bitter rejection response consists of a suite of withdrawal reflexes and negative responses. It is generally assumed to have evolved as a way to facilitate avoidance of food that are poisonous because they usually taste bitter to humans" (John I. Glendinning, *Is the Bitter Rejection Response always Adaptive?*, Physiology & Behavior, Vol. 56, N°6, 1994, p. 1217). No such evolutionary advantage has ever been demonstrated in the case of perception of consonance and dissonance.

[252] Remember that the term tonality is here used in the broad sense of relation with the diatonic scales, not in the historical sense that distinguishes between modal and tonal music. According to the latter, music from before the start of the seventeenth century can be called neither tonal nor atonal. For more details on the historical evolution of consonance and its perception, see for instance: Jacques Chailley, *Traité Historique d'Analyse Musicale*, Paris: Alphonse Leduc, 1951, pp.14-16.



Example 4.7: Historical evolution of consonances,
showing when the indicated intervals became accepted in triadic chords
(source: Jacques Chailley, *Traité Historique d'Analyse Musicale*, p. 15).

The role of cultural evolution in the perception of consonance is described by Gareth Loy as follows:

> Consonance appears to be influenced, but not determined, by underlying psychological principles we all share. It seems as well to be a matter of taste decided differently by each musical culture and each age. The harmonies in the chorales of J.S. Bach, for example, do not strike the modern ear as particularly dissonant; however, listeners of his age sometimes found them shocking. A similar progression has occurred with the music of Mozart, Beethoven, Wagner, Mahler, Debussy, Stravinsky, Schoenberg, among others. So where intervals are concerned, it seems that familiarity breeds consonance.[253]

Richard Parncutt addresses the acceptance of non-diatonic music that evolved in parallel with the evolution of consonance acceptance. He observed that "[t]he fact that many [non-diatonic] pieces have now been accepted into the concert repertoire supports the idea that any new style can be appreciated given sufficient exposure to that style, and provides evidence against the theory that diatonicism is in some way more "natural" than other ways of organizing pitch in music"[254]. The link between consonance and tonality will become clear in the discussion of the similarities between PC- and T-formulas.

With Arnold Schoenberg came the complete "emancipation of dissonance"[255], disconnecting consonance and dissonance entirely from triadic thinking. For twenty-first century people, a minor second is not less dissonant than it was for people during the Renaissance, but it is considered acceptable in a composition. For me at least a minor second doesn't sound less dissonant than it did when I was a child, as far as I can tell, but I have learned to appreciate its beauty. For me, and for many people with a similar cultural development, any collection of intervals is acceptable in a composition. The beauty of a composition is not determined by its level of consonance or dissonance, but by what is done with the tones and collections of tones that form the composition. A minor second may be an appealing interval to me (like I could have a preference for bitter food), but it can be very ugly when it is not used in a way I can appreciate in a composition. I would term certain dissonances in a piece in baroque style ugly and misplaced, but I would find the same dissonance incredibly

[253] Gareth Loy, *Musimathics, the mathematical foundations of music*, volume 1, MIT press, 2006, p. 60.
[254] Richard Parncutt, *Harmony, A Psychoacoustical Approach*, Springer Verlag Berlin, 1989, p. 6.
[255] Arnold Schoenberg, *Style and idea*, Leonard Stein, (ed.), Leo Black (transl.). Berkeley & Los Angeles: University of California Press, 1975, p. 260.

beautiful in another context. There is no single collection of tones that I could not find beautiful in the right context, even collections using other intervals than those of the Equal Temperament. If I restrict my research to Equal Temperament, it is because I have chosen to restrict my music to that tuning (I use quarter tones and other micro-intervals only to add colour to notes in Equal Temperament, and not in a structural way, as was discussed in Chapter 1).

Let me conclude this historico-aesthetic digression with the claim that, even if it is true that the perception of consonance has a physiological basis, this means little or nothing within the realm of aesthetics. The 'dictate' of the physiological basis would even constitute a significant inhibition for every aesthetic development. Culture is by definition in opposition with (though complementary to) nature. Every culture is different from human nature and the urge for cultural expression is part of human nature. A clear distinction has therefore to be drawn between nature and culture.

4.3.3 Objective criteria

If one wants to avoid the objections resulting from 'aesthetic' criteria for the definition of consonance and the determination of its degrees, one has to start from other criteria, criteria that do not involve (aesthetic) judgment since they are subjective. "Judgments of consonance also show effects of training, experience, development, and the particular methodology employed."[256] When we want to avoid those 'subjective' criteria for the determination of consonance or dissonance indices, the empirical results discussed above are inadequate. Other criteria have to be chosen, criteria that can be called 'objective'.

4.3.3.1 Tint affinity[257]

A candidate for a so-called 'objective' criterion is provided by Kolinski's 'tint-approach'. Kolinski calls "the quality that is identical in octave tones but is distinct in other tone relations" "tint". "The Western tone system", he says, "consists of a series of twelve different tints"[258]. He claims: "consonance is not fusion or homogeneousness in general but homogeneity of tints; accordingly, dissonance is heterogeneity of tints."[259] Consonance, he writes, is determined by 'tint affinity', "the degree of affinity between two tints [which] depends on their distance in the circle of fifths"[260]. "According to the quintal principle [the tint affinity-approach] the most consonant triads should be those formed by three tints adjacent in the circle of fifths"[261] (such as D-G-C).[262]

At first sight it looks like Kolinski's criterion of 'tint affinity' is based on a feature of the sounds that are combined, not on the perception or interpretation of those sound combinations. There are some problems with the criterion however. A first problem is that it makes a major second more consonant than a minor or major third,[263] a claim that is not supported by most experimental and theoretical studies or by common perception. An additional problem is that Kolinski claims "[t]here exists no direct tint affinity between tones separated by more than four steps in the circle of fifths"[264], which makes it unclear what the assumption of tint affinity is based on to begin with. It stays unclear what exactly this mysterious feature of 'tint affinity' refers to. Indeed, it is also unclear what is meant by the "quality shared by octave tones". Can it be measured objectively or is it only perceived—making it

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[256] Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* Oxford University Press, 1990, p. 54.
[257] Tint affinity is here described as only one of many possible 'objective' criteria. Again, it is not the objective of the present text to give a comprehensive overview of possible criteria.
[258] Mieczyslaw Kolinsli, *Consonance and Dissonance*, in: Ethnomusicology, Vol. 6, N°2, 1962, p. 67.
[259] Mieczyslaw Kolinsli, *Consonance and Dissonance*, p. 67 [author's original underlining left out].
[260] Mieczyslaw Kolinsli, *Consonance and Dissonance*, p. 68.
[261] Mieczyslaw Kolinsli, *Consonance and Dissonance*, p. 69.
[262] This would make fourth chords more consonant than major or minor triads, a claim that may well be justified.
[263] Kolinski states that some medieval theorists (e.g. Arezzo) also considered the major second as more consonant than the minor third (Mieczyslaw Kolinsli, *Consonance and Dissonance*, p. 68).
[264] Mieczyslaw Kolinsli, *Consonance and Dissonance*, p. 68.

subjective after all? Kolinski provides a hint for an answer to this question when he writes that "basic consonance results from tint identity and tint affinity and […] these phenomena evolve from the specific properties of the fundamental intervals octave, fifth and fourth due to the simplicity of their vibration ratios 1:2:3:4"[265].

Simplicity of frequency ratio's may indeed be a workable objective criterion for the determination of consonance indices, as will be discussed next, but simple frequency ratio results in a different order of consonance than tint affinity. As will be seen, it makes thirds more consonant than seconds, for instance. It is therefore unclear how tint identity and affinity 'result' from simple frequency ratios. This causal relation is also unclear in Kolinski's next claim that "there exists a close psychological parallelism between tint relations and vibration ratios. Consequently, we have to assume universal validity of the concept of basic consonance and dissonance"[266].

If 'tint affinity' results from simple frequency ratios, it is only a detour to arrive at the concept of consonance based on simple frequency ratios. Why then not use simple frequency ratios as the direct criterion for consonance?

4.3.3.2 Simple frequency ratios

The fact that some of the empirical dissonance indices discussed above are based on 'subjective' criteria is problematic because these subjective criteria may differ from individual to individual. Indeed, I for one consider ic6 to be more dissonant than ic 2, whereas the discussed dissonance indices ascribe a lower value to the dissonance index of ic 6, making it more consonant than ic 2. Not surprisingly, there is no consensus about the empirical dissonance indices for ic 6: as we have seen, Malmberg as well as Kameoka and Kuriyagawa's indices situate ic 6 between ic 2 and ic 3; Helmholtz's simple-ratio index and Hutchinson and Knopoff's index for ic 6 are comparable to that of ic 3, whereas Helmholtz's ET-index makes ic 6 considerably more consonant than ic 3—at the same level as ic 4—which does not reflect my personal perception.

The criterion of simple frequency ratios of 'pure' intervals[267], starting from the definition of consonance in terms of "the singular nature of tone intervals with frequency ratios corresponding with small integer numbers"[268]—as Greenwood and Plomp & Levelt state—proves to be a valuable 'objective' solution to the problem. Indeed, consonance is sometimes defined in terms of the 'simplicity' of frequency ratio's.[269] John Fauvel, for instance, claims: "Consonance is both a psychological and a physical criterion: two notes are *consonant* if they sound 'pleasing' when played together. In physical terms this seems to occur when the frequency ratio of the two notes is a ratio of low integers: the simpler the ratio, the more consonant are the two notes"[270]. In Fauvel's claim, the 'pleasing' character of consonant intervals *results* from the fact that the objective ratio of the frequencies of the tones in the interval is 'simple'; he does not *define* consonance in terms of the subjective pleasing character. The basis for his definition of consonance is, in other words, objective, not subjective. The (subjective) 'pleasing' character of consonant intervals is therefore irrelevant in the definition. As Max Meyer wrote: "All we know is that the degree of consonance depends in some manner upon the simplicity of numerical relations"[271], and that is all that is relevant in the present context.

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[265] Mieczyslaw Kolinsli, *Consonance and Dissonance*, p. 70.
[266] Mieczyslaw Kolinsli, *Consonance and Dissonance*, p. 70.
[267] The frequency ratio of an interval is the mathematical fraction that has the frequency of the highest tone of the interval as a numerator and the lowest tone as a denominator.
[268] Reinier Plomp & Willem J.M. Levelt, *Tonal Consonance and Critical Bandwidth*, in: Journal of the acoustical Society of America, Vol. 38, p. 548.
[269] For a review of theories on frequency ratio-based consonance, see: Reinier Plomp & Willem J.M. Levelt, *Tonal Consonance and Critical Bandwidth*, pp. 548-60.
[270] John Fauvel, Raymond Flood & Robin Wilson (eds.), *Music and Mathematics, From Pythagoras to Fractals*, Oxford University Press, 2003, p. 13.
[271] Max Meyer, *Contributions to a Psychological Theory of Music*, University of Missouri Studies, 1, 1901, p. 60.

Frequency ratio is an adequate criterion for an additional reason. I am not in the first place looking for a criterion for consonance that is consistent with a psycho-physiological explanation (because so far there is no consensus on such a criterion, although Fauvel suggests there is a causal link between simple frequency ratio and 'pleasantness' of perception), but for a criterion that is consistent with the way I approach consonance in my music: I regard consonance and dissonance as a feature of the sound combination, as "something inherent in the intervals"[272], resulting in a perception that may be different from individual to individual, not as a purely perceptive characteristic. In this sense, I consider consonance and dissonance as an acoustic quality. Therefore, my consonance indices have to be based on an objective criterion. Furthermore, I treat consonance and dissonance from an aesthetic point of view, not in the sense of aesthetic judgment, distinguishing between "consonant is pleasant" and "dissonant is unpleasant", but more generally in the sense of assessing how consonance and dissonance fit within my aesthetic idiom, within my aesthetic universe (see Chapter 7). Even if there is a causal relation between the simplicity of frequency ratios and the 'pleasing' perceptive character of consonance, and even if there seems to be—according to Fauvel—a factual relation between the two, this is only of secondary importance for my purpose.

4.4 Consonance indices, a mathematical model

The purpose of the present section is to theoretically determine consonance indices starting from the criterion of simple frequency ratios. These consonance indices will then serve as a basis to determine the degree of consonance of all possible pitch class sets (or set classes). As was the case in the discussion on tonality, consonance is a question of 'degree'. A sound combination is not either consonant or dissonant, but each sound combination, each pitch class set, has its degree of consonance. In this respect, I agree with Max Meyer, who "would outlaw the term 'dissonance' as having no scientific value, and speak merely of lesser degrees of consonance"[273].

The method used to determine consonance indices is not perfect; it has its weaknesses, just like any other method (including those of the classic studies). Gareth Loy shows how, for example, the approach dissonance metric of ET intervals based on critical bands is highly arguable, because with this approach "[t]he minor seventh and major second are predicted to be more consonant than the major third, for example. Also, it does not seem right that the tritone [ic 6] should have the same consonance as the major third"[274]. The aim of the calculations in the next section is to obtain consonance indices that are not only 'objective', but that correspond with physiological features of the auditory system, with general perception, and with my own (somewhat divergent) perception of ic 6.

4.4.1 Simple frequency ratio and ratio sum

Gareth Loy claims that if consonance of harmonic intervals is a feature inherent in the intervals "we should examine their mathematical properties"[275]. The frequency and wavelength—the wavelength of a tone being inversely proportional to its frequency—of the tones that form the interval are such mathematical property. Frequency ratios are therefore a convenient way to compare the frequencies of tones. Loy writes:

> Giovanni Battista Benedetti (1530-1590) is perhaps the first to relate pitch and consonance to frequencies of vibration. […] [h]e related interval consonance to the frequency of wave coincidence between two tones. He observed that an interval consists of a shorter wavelength (higher pitch) and a longer wavelength (lower

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272 Gareth Loy, *Musimathics, the mathematical foundations of music*, volume 1, The MIT Press, 2006, p. 56.
273 Martha Guernsey, *The rôle of Consonance and Dissonance in Music*, in *The American Journal of Psychology*, Vol. 40, N°2, 1928, p. 179.
274 Gareth Loy, *Musimathics,* p. 186.
275 Gareth Loy, *Musimathics,* p. 56.

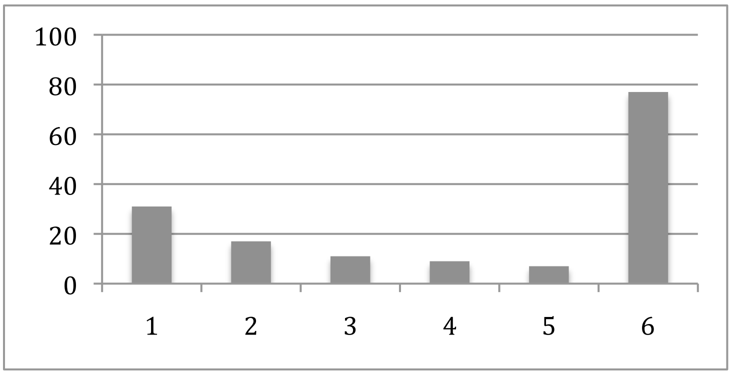
> pitch), and argued that the wavelengths of more consonant intervals coincide more often than do those of more dissonant intervals.[276]

This idea corresponds with the idea of the relationship between consonance and 'simplicity' of frequency ratios. If consonance of an interval or interval class[277] is measured on the basis of how 'simple' the frequency ratio of the frequencies of constituent tones is, it is important to define what it means for a frequency ratio to be 'simple'. Simplicity of frequency ratio can be defined on the basis of 'ratio sum'. The **ratio sum** of an interval is the sum of the numerator and the denominator of its frequency ratio. Example 4.8 lists the ratio sum of all interval classes in just intonation.

interval class	frequency ratio	ratio sum
ic 1	16/15	31
ic 2	9/8	17
ic 3	6/5	11
ic 4	5/4	9
ic 5	4/3	7
ic 6	45/32[278]	77

Example 4.8: Frequency ratios and ratio sums for all interval classes in just intonation.

Consonance of an interval class can then be defined as follows: the lower the ratio sum of the interval class, the more consonant the interval class. This works well for ic 1 through 5—as can be seen in Example 4.9—making ic 1 the most dissonant interval class and ic 5 the most consonant. The frequency ratios get 'simpler' from ic 1 to ic 5, and therefore the intervals get gradually 'more consonant' from ic 1 to ic 5. This corresponds with common perception, with my own perception, as well as with the 'classic' indices discussed above.



Example 4.9: Ratio sum (vertical axis) for all interval classes (horizontal axis) in just intonation.

The result for ic 6, however, does not fit the reality of common perception. Although there is, as we have seen, no consensus about the empirical dissonance index for ic 6, in none of the experiments ic 6 is perceived as being more dissonant than ic 1, let alone much more dissonant (77, the ratio sum for ic 6, is more than twice 31, the ratio sum for ic 1 in just intonation). A mathematical model for consonance that doesn't correspond to general perception (or at least with my own perception) is

276 Gareth Loy, *Musimathics,* p. 56.

277 Since I am looking for a way to quantify the degree of consonance of set classes, the calculations will be restricted to interval classes.

278 Ic 6 as a diminished fifth has a ratio sum of 64/45 in just intonation (see for instance: Ross W. Duffin, *How Equal Temperament Ruined Harmony (and Why You Should Care)*, W.W. Norton & Company, 2007, p. 163). 45/32 is the ratio of an augmented fourth (45/32). Clarence Barlow claims "theoreticians today usually allocate the ratio 32:45 to [ic 6]" (Clarence Barlow, *Two essays on Theory*, Computer Music Journal, Vol. 11, N°1, Microtonality, 1987, p. 46). In the present dissertation, this ratio is adopted.

irrelevant because it is merely theoretical.[279] Therefore adjustments are required. An additional shortcoming is that the frequency ratios in Example 4.8 apply to just intonation, whereas Equal Temperament (ET) is nowadays commonly used, especially in contemporary music. Eliminating this shortcoming may have an effect on the disproportioned value for the ratio sum of ic 6.

4.4.2 Equal Temperament and simplest frequency ratios

Equal Temperament divides the octave in "12 equal-sized semitones"[280]. The problem with this subdivision is that its intervals never result in simple frequency ratios. Indeed, "The frequency f_k of any equal-tempered interval k (= 0,1,…11) relative to reference frequency f_R"[281] is:

$f_k = f_R.2^{k/12}$

Therefore, the frequency ratio of interval class n in ET equals $2^{n/12}$ (where n is the interval class number (1 through 6)). The frequency ratios for all interval classes in ET are listed in the third column of Example 4.10 below. They are never fractions of integers. Therefore, "the equal-temperament scale in general use today does not consist of notes in exact simple ratios"[282].

interval class	frequency ratio just intonation	frequency ratio ET
ic 1	16/15	$2^{1/12}$ = 1,05946309436
ic 2	9/8	$2^{2/12}$ = 1,12246204831
ic 3	6/5	$2^{3/12}$ = 1,189207115
ic 4	5/4	$2^{4/12}$ = 1,25992104989
ic 5	4/3	$2^{5/12}$ = 1,33483985417
ic 6	45/32	$2^{6/12}$ = 1,41421356237

Example 4.10: Frequency ratios for all interval classes
in just intonation and ET

Groves Online Dictionary claims that this doesn't undermine sensory theories based on either beats or neural synchrony[283], because "the deviations from a simple ratio scale are small. For example, the interval of a perfect 5th corresponds to a frequency ratio of 3:2; on the equal-temperament scale the ratio is 2.9966:2. This deviation may produce a small increase in beating between the upper harmonics of complex tones, but the effect is not very noticeable."[284] Malmberg found that the difference between just intonation and ET did not result in a different order of consonance of intervals. Helmholtz ET results are not significantly different from his simple frequency ratio values either.

[279] It is important to distinguish between a concept of consonance based on subjective (psychological, perceptive,…) criteria and one that is based on objective (mathematical, physical,…) criteria that fits the reality of human perception. Searching for 'objective' dissonance indices that fit perception is not the same as basing the indices on subjective criteria.
[280] Gareth Loy, *Musimathics,* p. 39.
[281] Gareth Loy, *Musimathics,* p. 40.
[282] Groves online, lemma: consonance, https://stuiterproxy.kuleuven.be/subscriber/article/grove/music/,DanaInfo=www.oxfordmusiconline.com+06316?q=consonance&search=quick&source=omo_gmo&pos=1&_start=1#firsthit [last accessed: 21 February 2014].
[283] "Neural synchrony is the simultaneous / synchronous oscillations of membrane potentials in a network of neurons connected with electrical synapses (gap junctions). It is considered by some theorists to be the neural correlate of consciousness." (Robert Stufflebeam, *Neural Synchrony*, on: www.mind.ilstu.edu/curriculum/modOverview.php?mod GUI=233 [last accessed on 26 July 2014]). It may be considered to be a physiological characteristic.
[284] Groves online, lemma: consonance,https://stuiterproxy.kuleuven.be/subscriber/article/grove/music/,DanaInfo=www.oxfordmusiconline.com+06316?q=consonance&search=quick&source=omo_gmo&pos=1&_start=1#firsthit [last accessed: 21 February 2014].

Malmberg states: “when by the use of two sets of tuning forks, the just intonation was compared with the tempered intonation, no difference in ranking of the intervals large enough to affect the order resulted from the difference in temperament.”[285] Still, the formulation “not very noticeable” is vague, to say the least. John Fauvel’s claim that the concept of consonance should be broad enough “to admit that most of the so-called consonant intervals in our music are mistuned”[286] is also unsatisfactory in this respect. More precision and a substantiated justification for this claim are required. The extent to which ET may deviate from just intonation without requiring adaptations to the frequency ratios of interval classes has to be assessed. A solution to this problem is given by combining the physiological phenomenon of just noticeable difference for pitch difference discrimination and Clarence Barlow’s theory on “bending into place”.

4.4.3 Just noticeable difference

The human auditory system is limited in its ability to detect frequency differences. Joos Vos suggests that “scales in which the intervals were not more than 5 cents away from the ‘just’ version of the intervals […are] all close to equally acceptable”[287]. Pitch difference between two pitches can, in other words, only be perceived when the difference is larger than 5 cents. This is called the just noticeable difference (JND) for pitch difference discrimination. Richard Parncutt claims that for pure tones “a change can be heard in a pure tone if it is shifted in frequency by 0.06-0.12 semitones”[288]. This corresponds with a JND of 6 to 12 cents. In the following calculations, the lowest JND (5 cents) will be used. All claims valid for a 5 cent limit also apply to higher limits.

If a ‘simple’ ratio of two integers can be found for an interval in ET that lies within the 5 cents JND limit, no difference can be perceived between the interval represented by the simple ratio and the interval in ET. The simplest ratio (that is the ratio with the smallest ratio sum) can therefore be taken to represent the interval in ET.

In order to compare interval classes in ET and in just intonation, we determine the interval content in cents for all interval classes in both tunings in cents. The cent system “is a logarithmic scale in which there are 1200 cents to the octave. […] [E]ach semitone is 100 cents. […] To convert from a frequency ratio of r:1 to cents, the value in cents is”[289]:

$$1200.\log_2(r) = 1200.\ln(r)/\ln(2)$$

The values in cent for intervals in ET and just intonation, and their differences are shown in Example 4.11 below. Positive difference values indicate that the ET interval is larger than the interval in just intonation; negative values denote the opposite.

[285] Constantine Frithiof Malmberg, *The perception of consonance and dissonance*, in *Psychological Monographs*, Vol. 25, 1918, p. 107.
[286] John Fauvel, Raymond Flood, Robin Wilson (editors), *Music and mathematics, From Pythagoras to Fractals*, Oxford University Press, 2003, p. 85.
[287] David J. Benson, *Music, A Mathematical Offering*, Cambridge University Press, 2007, p. 16, referring to: Joos Vos, *Subjective acceptability of various regular twelve-tone tuning systems in two-part musical fragments*, Journal of the Acoustic Society of America, Vol. 83. N°6, 1988, pp. 2383-92. “Vos studied the sensitivity of the ear to the exact tuning of the notes of the usual twelve tone scale, using two-voice settings from Michael Praetorius’ Musæ Sioniæ, Part VI (1609). His conclusions were that scales in which the intervals were not more than 5 cents away from the “just” versions of the intervals […] were all close to equally acceptable, but then with increasing difference the acceptability decreases dramatically. In view of the fact that in the modern equal tempered twelve tone system, the major third is about 14 cents away from just, these conclusions are very interesting.” (David J. Benson, *Music, A Mathematical Offering*, pp. 16-7).
[288] Richard Parncutt, *Harmony: a psychological approach*, Springer Verlag Berlin, 1989, p. 27.
[289] David J. Benson, *Music, A Mathematical Offering*, p. 166.

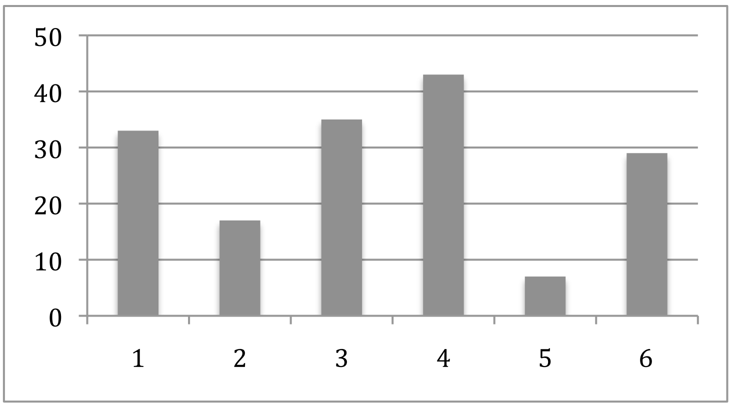
ic	ET	Just	difference
ic 1	100	111,73	-11,73
ic 2	200	203,91	-3,91
ic 3	300	315,64	-15,64
ic 4	400	386,31	13,69
ic 5	500	498,04	1,96
ic 6	600	590,22	9,78

Example 4.11: Cent values of interval classes in ET and just intonation compared.

Whenever the difference between the values in ET and Just Intonation is smaller than the JND limit of 5 cents, the difference between the interval classes in the two intonation systems will not be perceived. This is only the case for ic 2 and ic 5 (-3,91 and +1,96 respectively). The simple ratios 9/8 (for ic 2) and 4/3 (for ic 5) can therefore be adopted in ET. For the other interval classes however, the difference exceeds 5 cents. Therefore, JND alone doesn't account for the fact that we do not perceive the difference between intervals in just intonation and ET. For those interval classes, we have to look for the *simplest possible* ratio of two integers that lies within the JND limit of 5 cents. The simplest ratios within the 5 cents JND from ET are shown in the third column of Example 4.12 below. The second column of the table lists the interval content in cents of the interval classes with the simplest ratio within 5 cents from interval classes in ET. The third column lists the ratio sum of those interval classes. The graph of Example 4.13 visualizes the ratio sums.

interval class	cents	simplest ratio	ratio sum
ic 1	104,96	1,063 (17/16)	33
ic 2	203,91	1,125 (9/8)	17
ic 3	297,51	1,188 (19/16)	35
ic 4	404,04	1,263 (24/19)	43
ic 5	498,04	1,333 (4/3)	7
ic 6	603,00	1,417 (17/12)	29

Example 4.12: Simplest frequency ratios for all interval classes in ET (third column).
The interval content in cents of the intervals is listed in the second column,
the ratio sum in the fourth column.



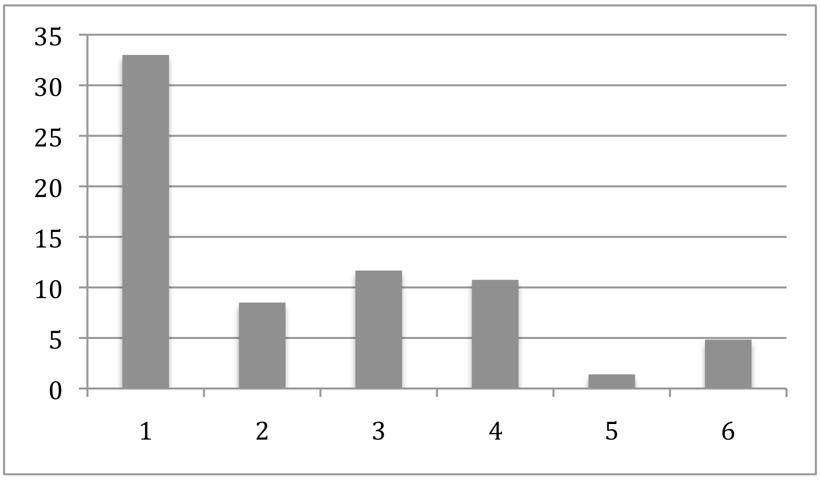
Example 4.13: ratio sum (vertical axis) for all interval classes
(horizontal axis) with simplest frequency ratios within 5 cents from ET.

As can be seen in Example 4.13, the value of the ratio sum for ic 6 is much more in accordance with the ratio sums for ic 1, 2 and 5, but the results for ic 3 and 4 are remarkably high; higher than that of ic 1, making ic 3 and 4 more dissonant than ic 1 if we use the 'simplest' ratio sums as consonance indices. This result does not fit general perception.

In an attempt to solve this problem, the values for the ratio sums may be divided by their interval class number. This can be justified by the idea that the size of an interval or interval class has an influence on its (perceived) consonance. Indeed, with increasing intervals, the influence of 'beats' decreases, and 'beats' are, according to Helmholtz, a cause of the perception of dissonance. But even dividing ratio sums by the corresponding interval class number does not yield a satisfactory result. The values for the adjusted ratio sums of ic 3 and ic 4 stay too high, as can be seen in Example 4.14 a and b. Only the values for ic 1, 2 and 5 show good correspondence with general perception as determined by Malmberg, Kameoka & Kuriyagawa, Hutchinson & Knopoff, Krumhansl, and Huron. Those results, of course, are for complex tones as compared to the pure tones (with no overtones) of the present calculations, but this cannot explain the big divergences.

interval class	ratio sum/ic number
ic 1	33
ic 2	8,50
ic 3	11,67
ic 4	10,75
ic 5	1,40
ic 6	4,83

a



b

Example 4.14 a and b: Ratio sum for simplest frequency ratios divided by interval class numbers for all interval classes.

4.4.4 Bending into place

If the perception of consonance were related to simple frequency ratios within the JND limit, then minor and major thirds should be perceived as being much more dissonant than they generally are, even if one takes into account the idea of decreasing dissonance with increasing interval size (division by the ic number). This is not in accordance with reality. Another correction has to be made to solve the problem for the frequency ratios of ic 3 and 4. The solution is provided by Barlow's idea of "bending into place". Clarence Barlow claims: "it is undisputable that a given interval with a complex numerical relationship in the direct vicinity of another, more harmonic interval, falls into the pull of the stronger one, as it were."[290]

According to Barlow, a "harmonic interval" is formed by two pitches with a simple frequency ratio. The degree of simplicity of a numeral relationship is what he calls "harmonicity". The table in Example 4.15 below lists all Barlow's harmonicity values higher than 0.05 within one octave. The higher the harmonicity, the higher the "pull". The harmonicity of 6/5 and 5/4 is very high (close to ±0,1). Therefore, Barlow claims, the "pull" of 6/5 (-0,099 for ic 3) and 5/4 (0,119 for ic 4) may be considered strong enough to bend ic 3 and ic 4 into place. This "bending into place" does not apply to the other interval classes because none of them has a simple ratio with high harmonicity in its vicinity, and they have ratios within the JND limit anyway.

Complete Intraoctavic Intervals upwards of Harmonicity 0.05

Interval-	Prime Decomposition as Powers of						Number-	
size (Ct)	2	3	5	7	11	13	ratio	Harmonicity
0.000	0	0	0	0	0	0	1:1	+∞
70.672	−3	−1	+2	0	0	0	24:25	+0.054152
111.731	+4	−1	−1	0	0	0	15:16	−0.076531
182.404	+1	−2	+1	0	0	0	9:10	+0.078534
203.910	−3	+2	0	0	0	0	8:9	+0.120000
231.174	+3	0	0	−1	0	0	7:8	−0.075269
266.871	−1	−1	0	+1	0	0	6:7	+0.071672
294.135	+5	−3	0	0	0	0	27:32	−0.076923
315.641	+1	+1	−1	0	0	0	5:6	−0.099338
386.314	−2	0	+1	0	0	0	4:5	+0.119048
407.820	−6	+4	0	0	0	0	64:81	+0.060000
427.373	+5	0	−2	0	0	0	25:32	−0.056180
435.084	0	+2	0	−1	0	0	7:9	−0.064024
470.781	−4	+1	0	+1	0	0	16:21	+0.058989
498.045	+2	−1	0	0	0	0	3:4	−0.214286
519.551	−2	+3	−1	0	0	0	20:27	−0.060976
568.717	−1	−2	+2	0	0	0	18:25	+0.052265
582.512	0	0	−1	+1	0	0	5:7	+0.059932
590.224	−5	+2	+1	0	0	0	32:45	+0.059761
609.776	+6	−2	−1	0	0	0	45:64	−0.056391
617.488	+1	0	+1	−1	0	0	7:10	−0.056543
680.449	+3	−3	+1	0	0	0	27:40	+0.057471
701.955	−1	+1	0	0	0	0	2:3	+0.272727
729.219	+5	−1	0	−1	0	0	21:32	−0.055703
764.916	+1	−2	0	+1	0	0	9:14	+0.060172
772.627	−4	0	+2	0	0	0	16:25	+0.059524
792.180	+7	−4	0	0	0	0	81:128	−0.056604
813.686	+3	0	−1	0	0	0	5:8	−0.106383
884.359	0	−1	+1	0	0	0	3:5	+0.110294
905.865	−4	+3	0	0	0	0	16:27	+0.083333
933.129	+2	+1	0	−1	0	0	7:12	−0.066879
968.826	−2	0	0	+1	0	0	4:7	+0.081395
996.090	+4	−2	0	0	0	0	9:16	−0.107143
1017.596	0	+2	−1	0	0	0	5:9	−0.085227
1088.269	−3	+1	+1	0	0	0	8:15	+0.082873
1129.328	+4	+1	−2	0	0	0	25:48	−0.051370
1137.039	−1	+3	0	−1	0	0	14:27	−0.051852
1200.000	+1	0	0	0	0	0	1:2	+1.000000

Example 4.15: Clarence Barlow's harmonicity list
(source: Clarence Barlow, *Mathematics as the Source of Music Composition*, p. 6
Presented at the MuSA Symposium, 02-08-2010 in Baden-Baden)[291].

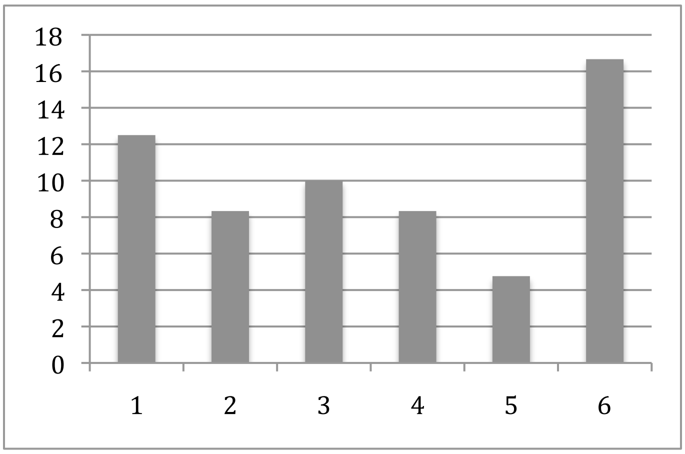
[290] Clarence Barlow, *Two essays on Theory*, in: Computer Music Journal, Vol. 11, N°1, Microtonality, 1987, p. 44. When asked what this claim is based upon, Clarence Barlow answered in an e-mail to me on 04-11-2010: "Being first and foremost a composer, it was a result of my musical intuition that led me to that conclusion. I have heard Mozart played on a slightly out-of-tune piano and that didn't change the meaning of the music. Also I have heard Bach played on a correctly tuned 12-tone equal tempered piano, and could enjoy the music even though a well-tempered piano might have been better. I can understand Americans, Australians and Brits speaking English, or Dutch and Flemings speaking Dutch. We always "bend" things to make them more meaningful." (quoted with kind permission of Clarence Barlow).

[291] Note that Barlow indicates intervals with inverted ratios, e.g. 5:6 for 6/5.

The idea of harmonicity alone doesn't provide a sufficient criterion for the determination of consonance indices. As can be seen in Example 4.16 and 4.17, the values for ic 3 and ic 4 are still problematic. Example 4.17 shows the inverse of the absolute values of harmonicities. This was done in order to make comparison with previously listed ratio sums easier. The result is comparable with the ratio sum for simplest frequency ratios divided by interval class numbers for all interval classes (see Example 4.14 b), except for ic 6, which shows the same deviation as the ratio sum in just intonation (Example 4.9).

interval class	ratio	harmonicity
ic 1	16/15	-0,076531
ic 2	9/8	+0,120000
ic 3	6/5	-0,099338
ic 4	5/4	0,119048
ic 5	4/3	-0,214286
ic 6	7/5	0,059932

Example 4.16: Frequency ratios and harmonicity values
for all interval classes after bending into place.



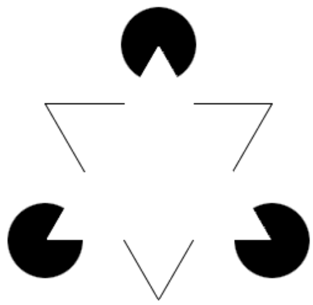
Example 4.17: Inverse of absolute values of harmonicity
for all interval classes after bending into place.

Although harmonicity doesn't provide an adequate remedy to solve the problem for ic 3 and 4 in ET, the idea of "bending into place" can be used to choose different simplest frequency ratios within the JND limit of an interval class. Indeed, it provides a justification for the choices of 6/5 for ic 3 and 5/4 for ic 4 as frequency ratios.

Bending into place is a process of involuntary correction by the human brain. It is therefore a physiological (objective) and not a subjective process. Donald Hodges and David Conrad Sebald claim: "The brain is an extremely efficient pattern of features detector. We want to make sense of things and to organize our surroundings."[292] According to Colin Ryan "[t]he Law of Prägnanz says that stimulus patterns are organized in the simplest way possible"[293]. In visual perception, for instance, the brain (involuntary) 'corrects' or 'constructs' visual perception.

[292] Donald Hodges & David Conrad Sebald, *Music in the Human Experience: An Introduction to Music Psychology*, Routledge, 2011, p. 130. Compare with Clarence Barlow's stated remark that "[w]e always "bend" things to make them more meaningful."
[293] Donald Hodges & David Conrad Sebald, *Music in the Human Experience,* pp. 130-1, referring to: Colin Ryan, *Exploring perception*, Brook/Cole, New York, 1997.

> Consider, for instance, the Kanisza figures (designed by Gaetano Kanisza, an Italian psychologist), which produce ‘subjective contours’—edges which are seen without a corresponding "objective" change in the visual field. Discussions of the figure [see Example 4.18] argue that the white triangle is "produced" by the viewer's response to the black elements in the configuration.[294]



Example 4.18: An example of a Kanisza figure
showing the construction of visual perception
(source: http://www.natureinstitute.org/txt/rb/art/perception.htm).

Mieczyslaw Kolinski describes this phenomenon as follows:

> [S]light changes of physical stimuli will hardly, if at all, alter corresponding sensations even if such deviations would mean substituting very complicated ratios for simple ones. For example, an ellipse with empirically equal axes will be perceived as a perfect circle in spite of the fact that the relation between major and minor axis could be expressed only through a quite complicated fraction; nevertheless, it is the approximation to the simple ratio 1:1 which causes the sensation of a circle. Similarly, slight deviations from theoretically exact sizes of consonant intervals, such as octave or fifth, would result in a change from simple vibration ratios, such as 1:2 or 2:3, to irrational fractions without affecting the specific character of these intervals. In such cases it is evidently the approximation to simple vibration ratios which prduces the sensation of consonance.[295]

This corresponds perfectly with Barlow’s idea of “bending into place”. Hindemith puts it this way:

> The distance between the two tones of [intervals other than octaves or fifths] can de diminished or augmented to a certain extent without destroying the impression of [the interval]. The slightest alteration in the size of an octave or a fifth, on the other hand, changes these intervals completely, so that the ear perceives them only as greatly expanded sevenths and fourths or greatly contracted ninths and sixths.[296]

4.4.5 Gravitational pull

What needs to be established in order to justify the choice of simplest frequency ratios is a manner to ‘weigh’ the ‘pull’ of harmonic intervals or the ‘gravity’ of frequency ratios. The simpler the ratio and

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[294] Ronald H. Brady, *Perception: Connections Between Art and Science*, The Nature Institute, online on: http://www.natureinstitute.org/txt/rb/art/perception.htm [last accessed: 28 July 2014].

[295] Mieczyslaw Kolinski, *Consonance and Dissonance*, in Ethnomusicology, Vol. 6, N°2, 1962, p. 70.

[296] Paul Hindemith, *Unterweisung im Tonsatz*, translated by Arthur Mendel as *The craft of Musical Composition* book 1, Theory, London, Schott & Co., 1942, p. 15. Hindemith, like many others, claims that ‘natural laws’ govern musical scales, and makes it seem as if the presence of octave and fifth in any scale is inevitable (because of this law). I disagree with this claim, but unfortunately, this discussion does not fall within the scope of the present dissertation.

the closer the interval to a harmonic interval, the greater its gravitational attraction (its pull). But how does simplicity relate to vicinity? How, for instance, can we decide whether to choose 16/15 or 17/16 as ratio for ic 1? 16/15 (1,0667) is simpler, and therefore has more gravitational attraction than 17/16, but 17/16 (1,063) is closer to 1,05946309436 (the ratio for ic 1 in ET) than 16/15 and lies within the JND limit.

I have developed the following formula to calculate the 'gravitational pull' of simple frequency ratio intervals on interval classes in ET:

pull of simple frequency ratio interval on interval class in ET = 'mass' / 'distance'

In this formula, 'mass' = 1 / ratio product of simple frequency ratio, and 'distance' = | ratio ET interval – ratio simple frequency interval |.

This formula provides a criterion for choosing simple frequency ratios. Although it does not reflect a physical reality[297], it has striking points of correspondence with Newton's formula for the force of gravitational attraction (F) between physical masses:

$F = G.m_1.m_2 / r^2$

where G is the gravitational constant, m_1 and m_2 are the masses of the two bodies and r is the distance between (the centre of gravity of) the two masses.

In the formula of gravitational pull of simple frequency ratio intervals, 'mass' is defined as the inversion of the ratio product (product of numerator and denominator of the ratio) of the simple interval. Indeed, the simpler the interval, the smaller its ratio product, but the higher its 'mass'. Therefore, 'mass' is defined as inversely proportional to ratio product. Calculations with the square of the 'distance' between ET and simple intervals (analogue to Newton's law) did not yield satisfactory results. Therefore the 'mass' of the simple interval is divided by the 'distance' between the simple interval and the ET interval, and not by the square of the 'distance'.

4.4.6 Consonance indices

The 'gravitational pull' formula can be applied to the frequency ratios of all interval classes in ET, in combination with the frequency ratios in just intonation and the simplest frequency ratios within the 5 cent JND limit. The result is shown in Example 4.19. The highest gravitational pull value determines the choice of frequency ratio (just intonation or simplest ratio) for interval classes in ET, because that is the harmonic interval the ET interval is bend into place to in our perception. These values are indicated in bold in Example 4.19.

interval class	ET ratio	just ratio	simplest ratio	just pull	simple pull
ic 1	1,05946309	16/15	17/16	0,57841672	**1,21059757**
ic 2	1,12246205	9/8	9/8	**5,472479614**	**5,472479614**
ic 3	1,18920712	6/5	19/16	**3,088454416**	-1,926919791
ic 4	1,25992105	5/4	24/19	**-5,039789189**	0,677506202
ic 5	1,33483985	4/3	4/3	**-55,31508845**	**-55,31508845**
ic 6	1,41421356	45/32	17/12	-0,087202738	**1,998268397**

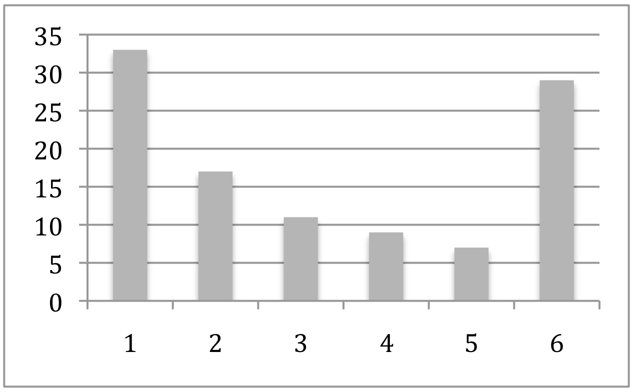
Example 4.19: 'Gravitational pull' of just interval and 'simplest ratio' intervals for all interval classes in ET (the strongest pull is indicated in bold).

[297] As will be discussed in Part 3, the formula describes an endophysical law of my personal aesthetic universe.

Since the frequency ratio with the strongest pull is the one the ET-interval will be bend into place to, it determines the choice between just intonation interval or interval with the simplest frequency ratio within the JND limit. The results are shown in Example 4.20 a, listing the simplest frequency ratios for all interval classes and their ratio sum.

interval class	simplest ratio	ratio sum
ic 1	1,063 (17/16)	33
ic 2	1,123 (9/8)	17
ic 3	1,2 (6/5)	11
ic 4	1,25 (5/4)	9
ic 5	1,333 (4/3)	7
ic 6	1,417 (17/12)	29

a



b

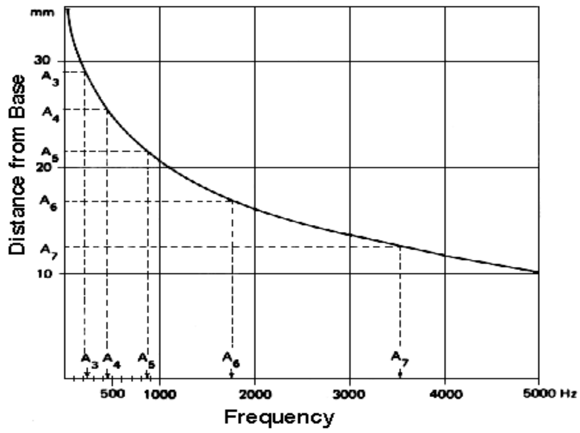
Example 4.20 a and b: Simplest frequency ratios and ratio sums
for all interval classes in ET after bending into place.

As can be seen in the graph of Example 4.20 b, the results are acceptable (and comparable to the 'classic' results) for ic 1 though 5. The value for ic 6 lies between that of ic 1 and 2, which is in accordance with my own perception, but is still too high. This problem can be solved the following way: if we take into consideration the fact that consonance increases with increasing interval size, not in a linear way (as we did before) but in a logarithmic way (because "the basilar membrane uses logarithmic encoding for pitch"[298]: "[i]f the frequency of a tone doubles, the position of maximum displacement along the basilar membrane moves toward the oval window by a constant amount."[299]), as shown in Example 4.21, we can divide the ratio sums in Example 4.20 a by $1+\log_{10}c$ (where c is the interval class number).

Note that the additional criterion is not subjective but both objective (interval size is a relation between the sounds involved) and (possibly) physiological (the way the auditory system functions). There is a probable correlation however between theory and physical reality; a correlation that is probably also more that coincidental in the 'classic' studies discussed above. As Carol Krumhansl remarks, the "substantial agreement" between the results of these studies "is true even though some of the sets of values are based purely on theoretical calculations and others are based on perceptual judgments"[300]. There is indeed reason to believe that perceptual judgment of consonance is influenced by physiological criteria and that these physiological criteria are described by theoretical formula and

[298] Gareth Loy, *Musimathics, the mathematical foundations of music*, volume 1, MIT press, 2006, p. 154.
[299] Gareth Loy, *Musimathics,* p. 153.
[300] Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* Oxford University Press, 1990, p. 59.

adaptations such as the logarithmic devision of ratio sums.[301]

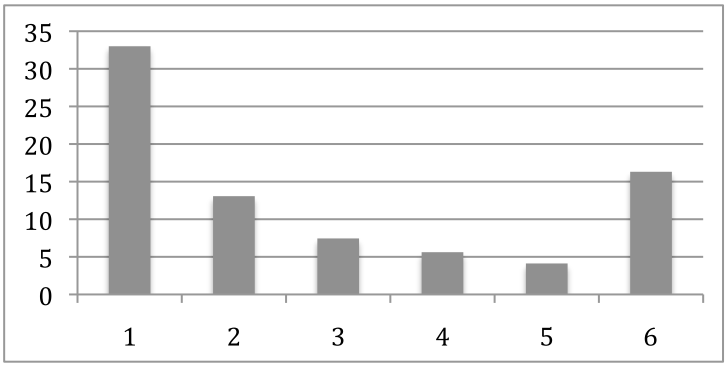


Example 4.21: Position of the resonance maximum of the basilar membrane for different frequencies (after Georg von Békésy, *Experiments in Hearing*, 1960, for a pure tone frequency f (linear scales) (source: Juan G. Roederer, *The Physics and Psychophysics of Music*, p. 32)[302].

This results in the theoretically obtained **consonance indices** for all interval classes listed and shown in Example 4.22 a and b.

interval class	consonance index
ic 1	33
ic 2	13,07
ic 3	7,45
ic 4	5,62
ic 5	4,12
ic 6	16,31

a



b

Example 4.22 a and b: Consonance indices for all interval classes in ET after bending into place and with logarithmic correction.

[301] The loragithmic division may me a guess (albeit an educated one). However, "Dirac discovered the correct laws for relativity quantum mechanics simply by guessing the equations. The method of guessing the equation seems to be a pretty effective way of guessing new laws." (Richard Feynman, *The character of physical law*, Penguin books, 1965 (1992), p. 57). Max Planck too produced a Kirchhoff distribution formula for the distribution of energies "by happy guesswork" (Eugene Hecht, *Physics*, Brooks/Cole Publishing Cie. 1994, p. 1048). As a matter of fact, the formulas developed in the present text all rely on some 'guesswork'. Some of it may be wrong, some may be good approximations or even 'lucky hits' based on intuition (for a definition of 'intuition': see Part 3), but even the inaccuracies or mistakes may be a proper basis for further developments. In other words, the theory I develop is heuristic, meaning it "serves as a guide in the solution of a problem but is otherwise itself unproved" (Eugene Hecht, *Physics*, p. 1052).

[302] Juan G. Roederer, *The Physics and Psychophysics of Music, An Introduction*, Springer Verlag, 4th edition, 2008, p. 32, referring to Georg von Békésy, *Experiments in Hearing*, McGraw Hill Book Company, New York, 1960.

Comparing Example 4.22 b with the classic consonance indices discussed earlier (see Examples 4.6 a-g)) shows that they are similar: there is a gradual decrease of degree of consonance from ic 1 to ic 5, which corresponds to that of the classic studies. As to the consonance index for ic 6: as we have seen, there was no consensus about its value in the classic indices. None of them made ic 6 more dissonant than ic 2. My result however situates the index for ic 6 between those of ic 1 and 2, which is in correspondence with the way I perceive sensory consonance. Therefore, the indices in Example 4.22 can be considered in accordance with both the commonly accepted indices and with my own perception and appear to be a valuable basis for the development of a formula to calculate the degree of consonance of any pitch class set.

To sum up, starting from an intuitive idea of consonance and dissonance in which I perceive ic 6 as being more dissonant than ic 2 and ic 3 (this is why ic 6 is the second most common interval in my dissonant idiom), and regardless of any existing definition for consonance or dissonance, I have developed consonance indices for sensory consonance that define my concept of consonance implicitly, and that are determined by the nature of the interval heard (its frequency ratio) as well as by the nature of the auditory sensory system (more precisely the basilar membrane). I do not claim that the physical aspects mentioned are truly the aspects that determine the human perception of consonance. There is only a striking resemblance between what I intuitively perceive as consonance and dissonance and the dissonance indices obtained with my method. As will be seen, this formula represents an endophysical law of my aesthetic universe.

4.5 Quantification of consonance of pitch class sets

4.5.1 Introduction

Most studies on consonance are limited to the consonance of intervals. As was stated before, Carol Krumhansl applies the concept of sensory consonance to "*pairs* of tones"[303]. The classic consonance indices discussed above too only relate to pairs of tones (intervals or interval classes). There is, however, no reason why the phenomenon of sensory consonance and dissonance could not be extended to larger combinations of pitches or pitch classes, in other words to pitch class sets.

There has been some research that assessed larger sets of sounds. In his unpublished master's thesis (1995)[304], Alan Smith takes the results of Malmberg (wrongly named 'Malberg' by Smith), Kameoka & Kuriyagawa, and Hutchinson & Knopoff as a starting point and extends their findings for pitch class sets consisting of more than two pitch classes. His does that in a "cumulative" manner, which "begins with a single tone and successively adds other tones. Each additional tone is chosen so that it exhibits the greatest amount of tonal consonance with the existing tones in the set"[305].

David Huron also calculates sensory consonance for sets with more than two pitch classes in a 1994 study[306]. "Huron's extensive study calculated the consonance in sets of 11 tones, sets of 10 tones, and so on, until all combinations of equally tempered tones were evaluated. The purpose of Huron's study, to identify the sets of tones offering an aggregate set of intervals that exhibit high tonal consonance, was quite different than the purpose of Krumhansl's comparison, which was concerned with key context stability"[307]. In the article on this study Huron claims: "Pitch-class sets (such as scales) can be characterized according to the inventory of possible intervals that can be formed by pairing all pitches

303 Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch,* Oxford University Press, 1990, p. 51 [my italics].
304 Allan B. Smith, *An alternative method of calculating tonal consonance in musical scales.* Unpublished master thesis, Massechusetts General Hospital Institute of Health Professions, Boston, 1995.
305 Allan B. Smith, *A "Cumulative" Method of Quantifying Tonal Consonance in Musical Key Contexts,* in Music Perception: An Interdisciplinary Journal, Vol. 15, N°2 (Winter, 1997), University of California Press., p. 180.
306 David Huron, *Interval-Class Content in Equally Tempered Pitch-Class sets*: *Common scales exhibit optimum tonal consonance*, in *Music Perception: an Interdisciplinary Journal*, Vol. 11, N°3, 1994, pp. 289-305.
307 Allan B. Smith, *A "Cumulative" Method of Quantifying Tonal Consonance in Musical Key Contexts,* p. 179.

in the set. The frequency of occurrence of various interval classes in a given pitch-class set can be correlated with corresponding measures of perceived consonance for each interval class."[308] He continues:

> In order to explore the relationship between pitch-class sets and perceived consonance, two indices are required: (1) a way of characterizing the frequency of occurrence of various intervals for any given pitch-class set, and (2) an index of tonal consonance for intervals of various sizes. By relating the number of possible intervals of each size with the corresponding tonal consonance of each interval, it is possible to determine the degree to which a given pitch-class set maximizes the potential for consonant-sounding intervals.[309]

These ideas lie at the basis of the construction of the formula to calculate degrees of consonance of pc-sets; the first requirement was fulfilled by calculating consonance indices for all interval classes, the second requirement will be part of the construction of the formula.

4.5.2 Prime consonance

The consonance indices for interval classes were determined on the basis of the frequency ratios of the smallest interval belonging to each of the six interval classes (the basic interval). The consonance index of ic 1, for instance, was determined with the 'simplest' frequency ratio of the minor second. The minor second is only one of the members of the class however, and different members of an interval class usually have different perceived levels or degrees of consonance.

Indeed, Smith Brindle—as was mentioned—claims that inversions of the three basic consonant intervals (fifth, major third and minor third) are less consonant than the original interval. In general, octave displacement influences the degree of consonance of an interval. The greater the pitch distance between the two pitches, the less they will be perceived as dissonant. A minor second is generally perceived as a stronger dissonance than a major seventh or a minor ninth, for instance.

> Of course interval size alone does not entirely account for the perceived consonance of two concurrent tones. […] spectral content, sound pressure level, and pitch register are known to affect the perception of tonal consonance. In order to compare interval-class inventories with tonal consonance, we must assume spectral content, sound pressure level, and pitch register do not vary systematically with the type of pitch-class set. Expressed more formally, we must assume that all music generated from the various pitch-class sets are played by similar instruments, at roughly the same loudness, in approximately the same pitch region.[310]

There is therefore no single degree of consonance of a pitch class set, but only for pitch intervals—and even then, tone colour, dynamics, or register (may) have an influence on the perceived degree of consonance of a harmonic sound compound. The degree of consonance of a pitch class set will therefore be defined as the degree of consonance of the prime form of the set class the pc-set belongs to, regardless of tone colour, dynamics or register. It will therefore be called the degree of **prime consonance** of the pitch class set or the set class[311]. Although the concept of consonance is thereby turned into a theoretical concept, it still has a perceptual link.

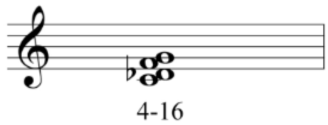
The fact that tone colour is discarded in the determination of prime consonance is the reason why the (theoretical) classic consonance values that are based on beats between partials, such as those of

[308] David Huron, *Interval-Class Content in Equally Tempered Pitch-Class sets*, p. 289.
[309] David Huron, *Interval-Class Content in Equally Tempered Pitch-Class sets*, p. 291.
[310] David Huron, *Interval-Class Content in Equally Tempered Pitch-Class sets*, p. 292.
[311] Degrees of consonance of pitch class sets or set classes will be used equivalently. It should be clear from the context whether pc-sets or whole set classes are meant.

Helmholtz, or those of Hutchinson & Knopoff (based on an extension of the Helmholtz - Plomp & Levelt model of beating as the cause of dissonance), are inadequate for the present purpose. The consonance indices of some of the other classic studies (Malmberg, and Kameoka & Kuriyagawa) result from experimental data based on psychological or perceptive criteria; they are subjective, and were therefore considered inadequate before. An additional problem with experimental methods to determine consonance indices is that they have to be performed very cautiously to avoid interpretations of relative musical consonance. When chords (tone compounds) in an experiment are played in series, judgment may be influenced by the chords heard before, thus resulting in judgment of musical rather than sensory consonance. This is why it was necessary to construct other consonance indices that respond better to the present requirements. These indices are used to determine the degree of prime consonance of all pitch class sets with the formula developed next.

4.5.3 Construction of the PC-formula

The degree of consonance of a pitch class set based on consonance indices for individual interval classes can be determined starting from the interval vector of the (set class of the) pc-set. The interval vector of a set class is the ordered set of 6 numbers indicating the number of times the interval classes 1 through 6 (in that order) appear between the elements of the set class. Pc-sets belonging to set class [4-16] (see Example 4.23 a), for instance, contain one instance of ic 1, ic 2, ic 4, and ic 6, two instances of ic 5 and none of ic 3, as shown in Example 4.23 b. Its interval vector is therefore <110121>.



Example 4.23 a: Prime form of set class [4-16] (represented by its prime form).

Example 4.23 b: Interval classes contained in set class [4-16] (represented by its prime form).

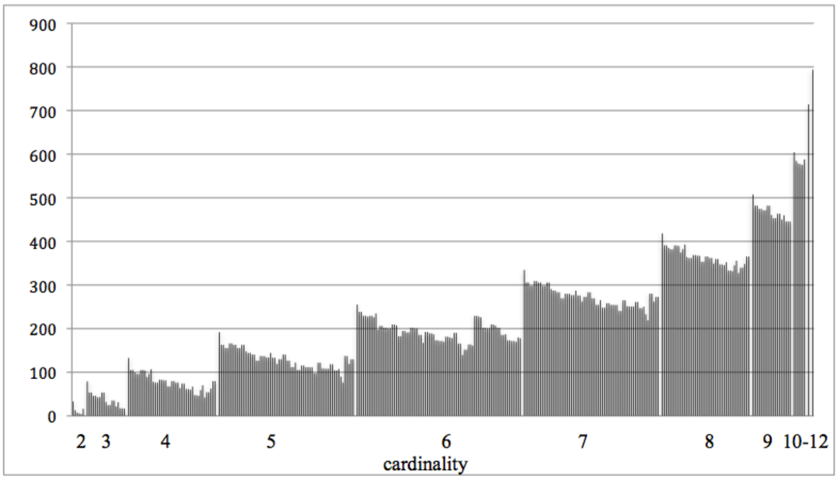
The first step in the construction of a formula for the determination of the degree of prime consonance of pitch class sets—henceforth called **PC-formula**[312]—consists of the summation of each number in the interval vector with the corresponding consonance index (Example 4.22) as given in Formula 1.

$$\sum (n_i . I_i) \qquad \text{(Formula 1)}$$

In Formula 1, n_i indicates the number of interval classes i in the pc-set and I_i the consonance index for interval class i.

For set class [4-16] this would give: 1x33 + 1x13,07+ 0x7,45+ 1x5,62 +2x4,12 +1x16,31 = 76,24
Results for all possible set classes are indicated in the diagram shown in Example 4.24.[313]

[312] PC stands for "prime consonance". It is written in capitals to distinguish it from the abbreviation 'pc' for 'pitch class' in pc-set.
[313] Note that set classes and their inversion always have the same degree of prime consonance because they have the same interval vector. This is also the case for Z-related set classes.



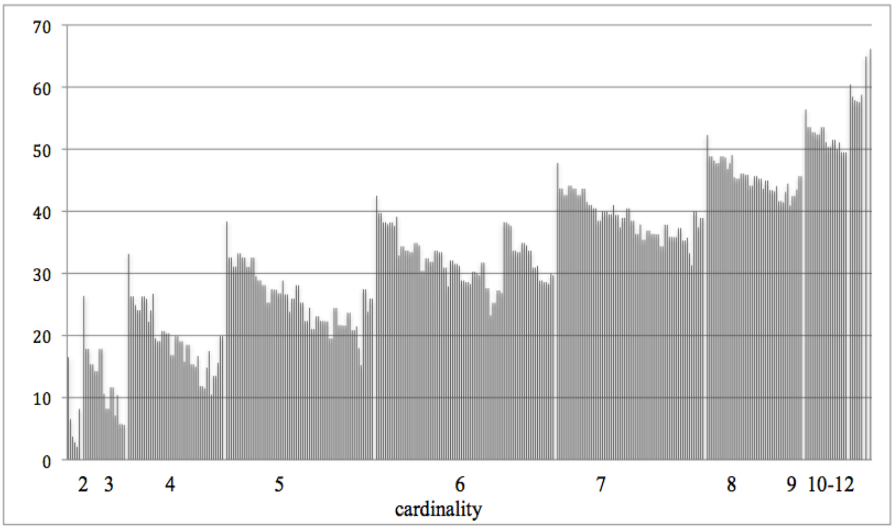
Example 4.24: Dissonance values (vertical axis) for Formula 1 for all set classes ordered by cardinality (horizontal axis).

The diagram shows a clear separation of (provisional) degrees of consonance[314] for the different cardinality groups; virtually all members of a cardinality group have a lower 'degree of consonance' (a higher dissonance value according to Formula 1) than all elements of a lower cardinality group. The mere fact that a pc-set contains more elements cannot be enough to make its degree of consonance lower than any pc-set with less elements, because this does not correspond with how consonant pc-sets are actually perceived. To cancel this effect the values of Formula 1 are divided by the cardinality of the pc-set in a next step:

$$\sum (n_i \,.\, I_i) / c \qquad \text{(Formula 2)}$$

where c is the cardinality of the pc-set.

The (dissonance) values obtained with Formula 2 are shown in the diagram of Example 4.25:



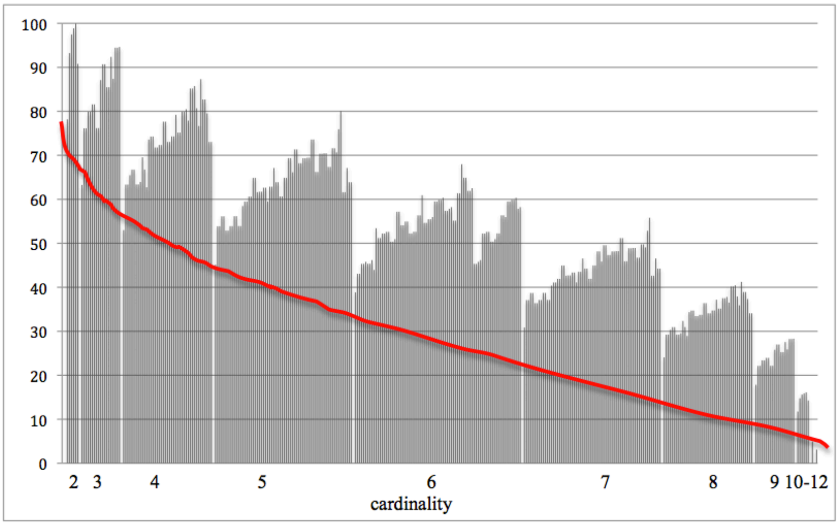
Example 4.25: Dissonance values (vertical axis) for Formula 2 for all set classes ordered by cardinality (horizontal axis).

In order to obtain values between 100 (for set class [12-1]) and the lowest value for ic 5 (the most consonant interval class) whilst preserving the same proportions between all the values, every value is

[314] The values are actually not indications of degrees of consonance but degrees of dissonance.

multiplied by a factor w = 100 / 66,1433; 66, 1433 being the highest value of Formula 2 (for set class [12-1]). This results in a value of 3,11 for ic 5. To change degrees of dissonance into degrees of consonance the diagram is then 'turned upside down' by subtracting every value from 103,11, in order to obtain the highest value (100) for ic 5 (103,11 - 3,11 = 100). This adaptation of the formula is only a way to rescale the values; it is no essential change to the values. The rescaling results in the following formula (Formula 3) and diagram (Example 4.26):

$$103{,}11 - [\,(\sum (\, n_i \,.\, I_i \,) / c\,) \,.\, w] \qquad \text{(Formula 3)}$$



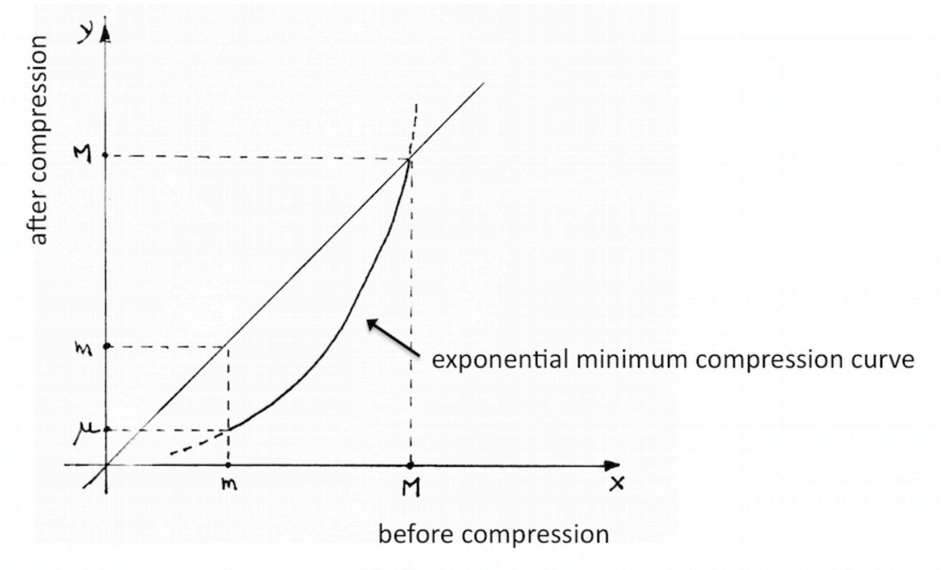
Example 4.26: Consonance values (vertical axis) for Formula 3 for all pitch class sets ordered by cardinality (horizontal axis).

The red curved line in the diagram in Example 4.26 connects the values for the lowest degree of consonance within each cardinality group. Those values are too high. The value for ic 1 (minor second), for instance, is higher than the values for most pitch class sets with four elements that are perceived more consonant; the value for ic 1 (78,16) obtained with Formula 4 is comparable to that of a dominant seventh chord (set class [4-27i], with value 82,69) or diminished seventh chord (set class [4-28], with value 79,52). To correct this, a minimum compression correction is made to Formula 3 in order to reduce the values proportionally and bring the minima to the expected values, whilst preserving the maxima, as was done in the construction of the T-formula. In the case of the T-formula, a logarithmic minimum compression function $\mathcal{K}$(x) was applied (see Formula 7 in Chapter 3); for the PC-formula, the exponential minimum compression $\mathcal{L}$(x) (Formula 4) proved to yield the best results.[315]

$$\mathcal{L}(x) = M \,.\, e^{[\ln(\mu / M)\,(x - M)/(m - M)]} \qquad \text{(Formula 4)}$$

The curve for this function is shown in Example 4.27.

[315] This result was obtained by trial and error. First a linear minimum compression was tried with different slope values. This did not provide a credible distribution. An exponential function produced better results. Several values for μ were tried, and implemented in analysis. The values used in Formula 5 proved to be the best value because they have the best distribution and they keep the PC-value of chromatic CIG's (see later) under 50 (a psychological border more than a theoretical one).

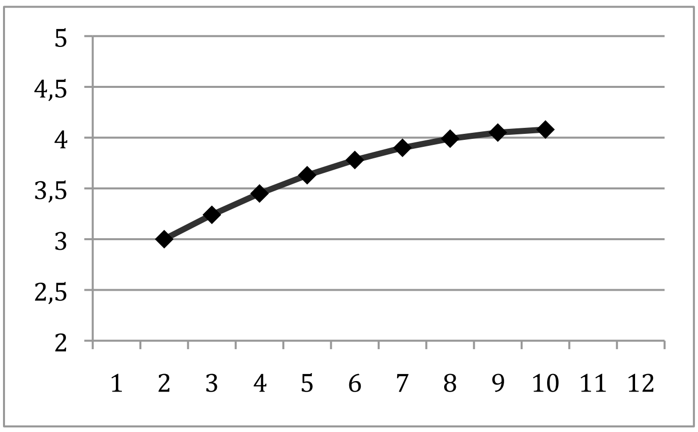


Example 4.27: Exponential minimum compression curve.

In Formula 4, M stands for the maximum value for Formula 3 within the cardinality group of the pc-set (or set class) in question; m is the minimum value; the value of μ, the minimum value after compression, is the minimum m of the cardinality group divided by a denominator d that evolves with the cardinality of the group in the following way: d starts at 3 for set classes with cardinality 2; in symbolic notation: d(2) = 3. For higher cardinalities (between 3 and 10):

$$d(c) = d(c-1) + [\ 3\ .\ (11- c\ )\ \ /\ 100\ ] \qquad \text{(Formula 5)}$$

The graph of Formula 5 is shown in Example 4.28. The table of Example 4.29 lists the values of M and m for cardinalities 2 to 12 obtained with Formula 3, as well as the values for d and μ for cardinalities 2 to 10. Since set classes [11-1] and [12-1] are both minima and maxima within their cardinality group, a minimum compression is not possible. The pc-values have been reduced to 2 for set class [11-1] and 1 for set class [12-1], the most dissonant set class.



Example 4.28: Values for d (vertical axis)
for all cardinalities (horizontal axis).

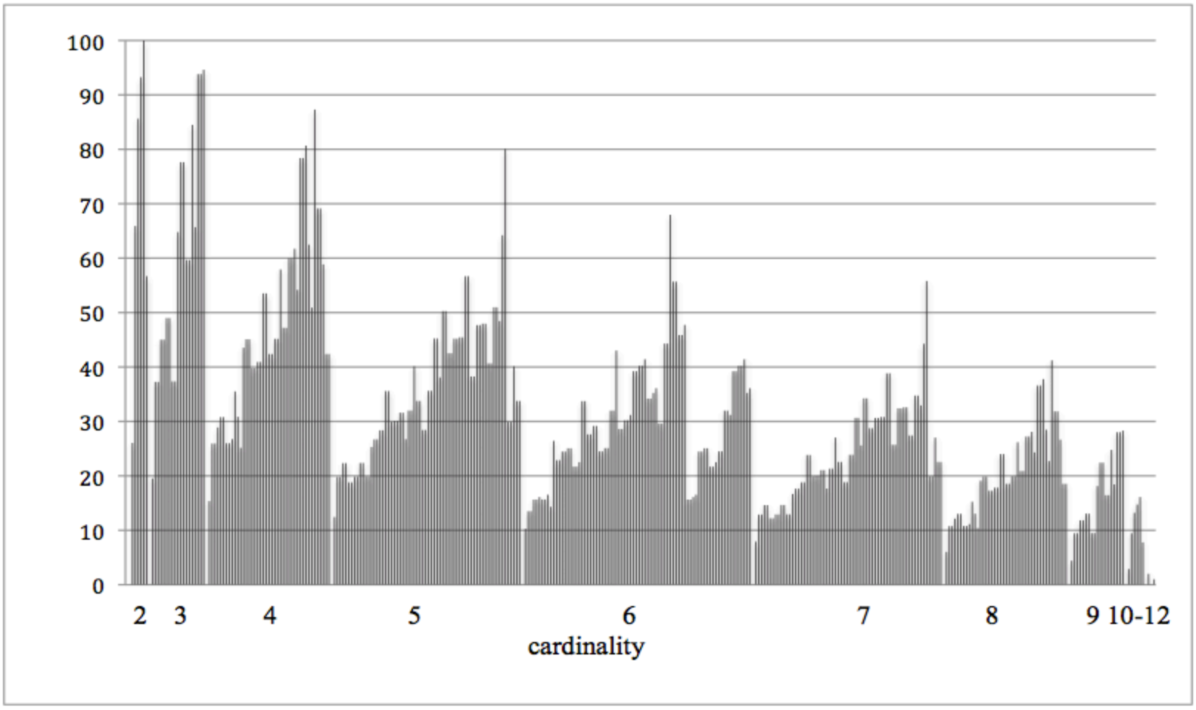
c	M	m	d	µ
2	100	78,16	3,00	26,05
3	94,61	63,26	3,24	19,47
4	87,30	53,00	3,45	15,36
5	80,07	45,14	3,63	12,43
6	67,97	38,86	3,78	10,28
7	55,80	30,85	3,90	7,91
8	41,22	24,07	3,99	6,03
9	28,30	17,85	4,05	4,41
10	16,11	11,74	4,08	2,88
11	4,96	4,96	-	-
12	3,11	3,11	-	-

Example 4.29: Values for M, m d and µ for all cardinalities (c)
obtained with Formulas 3 and 5.

Applied to Formula 3, the exponential minimum compression function $\mathcal{L}(x)$ results in the 'final' version of the PC-formula shown in Formula 6. A list with all prime consonance values can be found in Appendix 1. A graph is shown in Example 4.30.

PC-Formula:

$$\boxed{P_{(c\text{-}n)} = \mathcal{L}(103{,}11 - [\,(\textstyle\sum (\, n_i \,.\, I_i \,) / c \,) \,.\, w])} \qquad \text{(Formula 6)}$$



Example 4.30: Prime consonance values (vertical axis)
for all set classes ordered by cardinality (horizontal axis).

4.6 Applications and consequences of the PC-formula

4.6.1 Prime consonance analysis technique

The PC-formula can be used as a basis to develop a **Prime Consonance Analysis Technique**, or **PCA-technique**. This procedure is not as complex as the development of the TA-technique described in Chapter 3. Since PC-analysis deals with sensory consonance, and not with musical consonance, there is no need to take into account the context (the chords or tone complexes preceding the one in question) as is the case in the TA-technique. PC-analysis results in graphs showing the evolution of prime consonance of the analysed pieces.

To illustrate the PCA-technique, let us return to the pieces that were analysed in the discussion of T-analysis. The first analysed piece (see Section 3.5) was the *Aria* from Johann Sebastian Bach's *Goldberg Variations* BWV 988.

Example 4.31: Johann Sebastian Bach, *Aria* from *Goldberg Variations* BWV 988, bar 1-4.

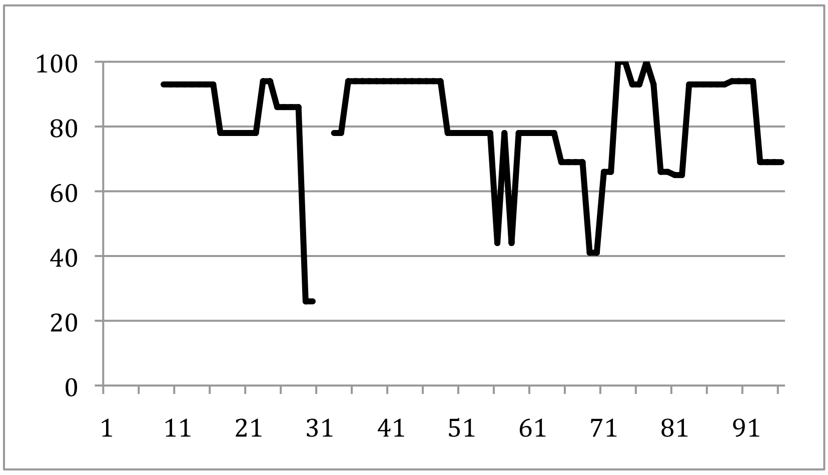
The first four bars of the score are shown again in Example 4.31 (see also Example 3.28). The *Aria* starts with a G (in double octave), which represents a single pitch class. The concept of sensory consonance does not apply to single pitch classes ($PC_{(1\text{-}1)}$ = undefined). On the second beat a B is added, producing a minor sixth + octave interval, which is an instance of set class [2-4] with PC-value $PC_{(2\text{-}4)} = 93$. On the last beat of the first bar, when A and D are added to the sustained G and B, an instance of [4-22] is sounded. $PC_{(4\text{-}22)} = 78$. On the last semi quaver of the bar, the 'dissonant' A is 'resolved' in B, turning the pitch class set in a highly consonant major triad (an instance of [3-11]) with $PC_{(3\text{-}11)} = 94$.

At the onset of bar 2, ic 3 (set class [2-3]) is heard. $PC_{(2\text{-}3)} = 86$. This means that the degree of prime consonance decreases. One might rightly object that a minor third cannot be less consonant than a major triad, which contains ic 3 and an additional ic 4 and ic 5. This is the result of the fact that the cardinality of a pc-set is taken into account in the determination of degrees of prime consonance. Therefore, in PC-analyses, one has always to interpret the results with this idea in mind. The fact that [2-4] and [2-5] (ic 4 and 5) have a higher degree of consonance than [2-3] (ic 3) makes [3-11] more consonant than ic 3.

On the second half of the second beat of bar 2, an instance of ic 2 is heard (F sharp and G sounding simultaneously). This set class ([2-1]) is highly dissonant. Its PC-value is $PC_{(2\text{-}1)} = 26$. Big drops in the degree of prime consonance happen, even in highly consonant and tonal music such as Bach's, at the occurrence of resolution-based musical dissonants, but they are always followed by a 'resolution', which constitutes a restoration of the high degree of prime consonance. This happens after a short octave F sharp, which is the actual resolution of G, but which has no degree of prime consonance ($PC_{(1\text{-}1)}$ = undefined). Still, the high degree of prime consonance is restored in the subsequent major triad F sharp-A-D with $PC_{(3\text{-}11i)} = 94$, after a short and more dissonant F sharp-A-E ($PC_{(3\text{-}7)} = 78$). Also, the notes of the dissonance F sharp-G are not 'attacked' simultaneously, which softens the perceived dissonance, an element that is not taken into account in the PCA-technique. If it were, the degree of consonance would not drop as steeply in absolute values as it does now, but the tendency would stay the same.

On the first beat of bar 3, the score shows an instance of ic 3, but since the G is ornamented with a trill and mordant (G-A trill followed by F sharp), the sound complex is interpreted as [3-7] (E-G-A) and brief 'additional' F sharp, turning it into [4-10]. The PC-values are $PC_{(3-7)}$ = 78, and $PC_{(4-10)}$ = 44 respectively. Strictly speaking, the four pitch classes of [4-10] never sound together, but musical ornaments are meant to create some kind of tension and should therefore, in my opinion, be interpreted as sound complexes in PC-analysis.[316] The second beat of bar 3 is similar to the first: a 'double cadence' consisting of a trill G-A on top of the sustained E, with a short touch of F sharp at the start, yielding the same PC-values as the previous beat. The tension is still not resolved in the next beat, featuring consecutively [4-27] (E-G-A-C sharp), [4-13] (E-F sharp-G-C sharp), and [3-10] (E-G-C sharp), with PC-values $PC_{(4-27)}$ = 69, $PC_{(4-13)}$ = 41, and $PC_{(3-10)}$ = 66. The resolution of this bar full of musical tension comes on the downbeat of bar 4 (interval D-A) as is required in common practice tonal music: $PC_{(32-5)}$ = 100. Figurative notes (passing notes and appoggiatura) drop the degree of prime consonance only slightly on the last semiquaver of the first beat (with the lowest value $PC_{(2-2)}$ = 66). At the very end of the bar, new musical (functional) tension and lower sensory consonance is created by the introduction of a dominant seventh chord [4-27] ($PC_{(4-27)}$ = 69). This tension is resolved in the next bar.

PC-analysis results in a PC-graph, showing the evolving degree of prime consonance of the analysed piece. Example 4.32 shows the PC-graph for the first four bars of Bach's, *Aria* from the *Goldberg Variations*. Note the 'gaps' in the graph when single sounds (or octaves) occur in the music (gaps would also occur on rests, when no sound is produced at all). Note also the restoration of a high degree of prime consonance after more dissonant moments. And note the fact that low degrees of prime consonance are never sustained for a long time and are only transitory in highly consonant music.

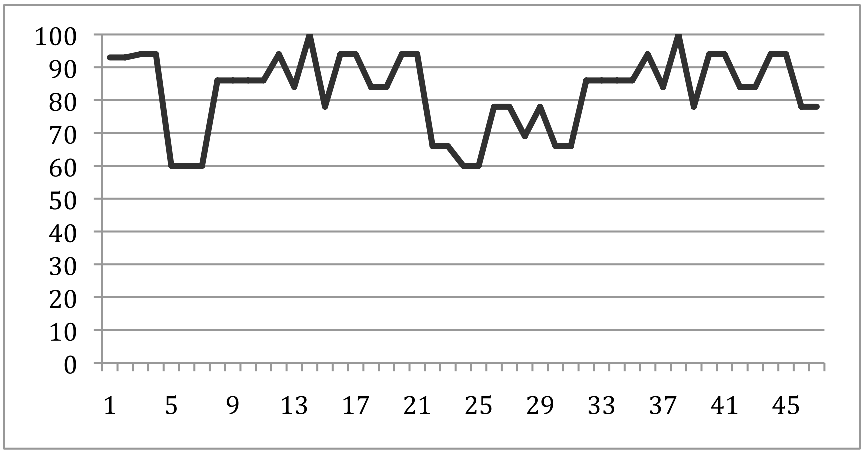


Example 4.32: PC-graph of Johann Sebastian Bach's, *Aria* from *Goldberg Variations* BWV 988, bar 1-4.
(unit on horizontal axis: demi-semiquaver)

The first four bars of the *Aria* have an average degree of prime consonance of $PC_{(av)}$ = 81,27.

A T-analysis was performed on the 25th *Goldberg* variation as an example of highly chromatic tonal music in Chapter 3 (see score in Example 3.37). A PC-analysis of the first two bars of the piece yields the PC-graph shown in Example 4.33.

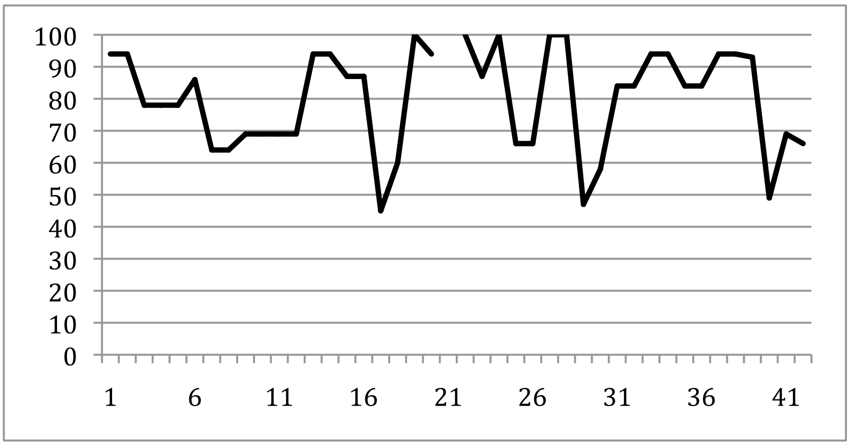
[316] In numerical value, the alternative interpretation of the trill G-A as swiftly alternating ic 1 and 2 would result in a fast skipping between $PC_{(1-1)}$ = undefined and $PC_{(2-2)}$ = 66, ending with a brief G-F sharp, $PC_{(2-1)}$ = 26, which would constitute no essential difference.



Example 4.33: PC-graph for bars 1 and 2 of Johann Sebastian Bach's *Variation 25* from *Goldberg Variations* BWV 988 (unit of the horizontal axis = demi-semiquaver).

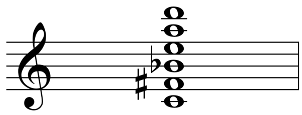
The average degree of prime consonance of the analysed bars is $PC_{(av)}$ = 82,65, which is almost the same as that of the first four bars of the *Aria*. Comparing Example 4.32 and 4.33 shows that, although the PC-curve of the *Aria* reaches lower degrees of prime consonance, it also stays on the maximum level (100) longer. Conversely, the PC-curve of *Variation 25* only rarely reaches PC-value 100 but never drops lower than 60.

The PC-graph of bars 18 to 21 of the prelude to Wagner's *Tristan and Isolde* (score in Example 3.45) is shown in Example 4.34. With its average degree of prime consonance $PC_{(av)}$ = 80,14 it is not significantly more dissonant than the *Goldberg Variations* (although it is less tonal). There are more fluctuations than in the *Aria* (due to the use of more different kinds of chords (set classes) in highly chromatic music), but the consonance range is comparable, even to that of the highly diatonic *Aria*. The degree of prime consonance of the analysed bars from the *Tristan and Isolde* Prelude fluctuates between 100 and 45 (for pc-set E-F-A-B, an instance of set class [4-16] on the fifth beat of bar 19). The lowest value isn't even as low as the 26 ($PC_{(2-1)}$) for the momentary passing ic 1 (F sharp - G) in bar 2 of the *Goldberg Aria*.



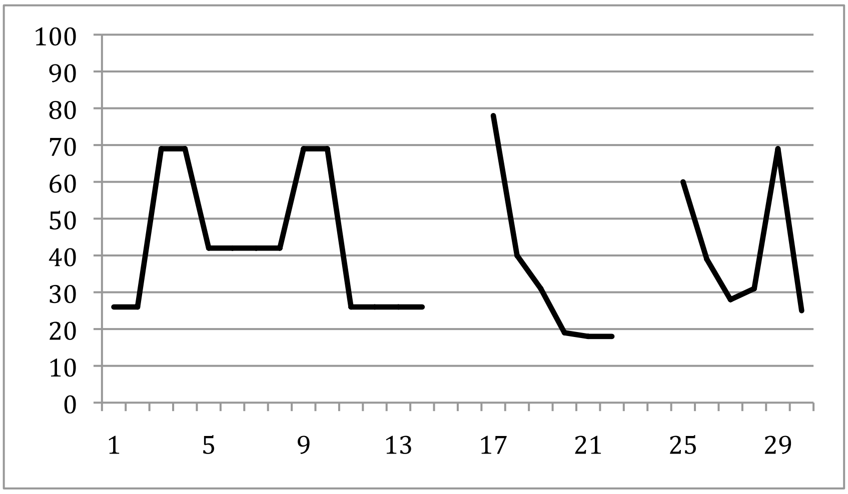
Example 4.34: PC-graph of the prelude to Wagner's *Tristan and Isolde,* bar 18-21 (unit of the horizontal axis = semiquaver).

Wagner's music illustrates the evolution in tonality (due to increased chromaticism), but not in consonance. This is due to the fact that late romantic composers such as Wagner used the same chords as the composers that preceded them: chords that are built up with superimposed thirds above the root note. The alternative chord formation of quartal chords, in which interval of fourths replace the thirds (used by composers such as Alexander Scriabin, Hindemith or the early Schoenberg) only slightly causes a decrease of the degree of prime consonance. Indeed, a triad formed by superimposing two perfect fourths still has a high degree of prime consonance of $PC_{(3\text{-}9)} = 84$; three superimposed perfect fourths yield $PC_{(4\text{-}23)} = 81$; even quartal chords (with perfect fourths only) containing five pitch classes ([5-35]) still have a high degree of prime consonance: $PC_{(5\text{-}35)} = 80$. The degree of prime consonance only starts to decrease significantly with quartal chords containing six pitch classes (five superimposed perfect fourths): $PC_{(6\text{-}32)} = 68$. Six note chords like Scriabin's *Mystic chord* (Example 4.35), an instance of [6-34], containing also augmented and diminished fourths, have a lower degree of prime consonance, of course ($PC_{(6\text{-}34)} = 46$).



Example 4.35: Scriabin's *Mystic chord*.

Significant change in degrees of prime consonance occurs by the turn of the twentieth century with the work of composers such as Arnold Schoenberg, who look for non-standard chord formations. Only with this abandonment of traditional triadic (or quartal) thinking came the complete "emancipation of dissonance"[317]. When chords can be constructed completely independent of triadic thinking, low degrees of prime consonance can be achieved. The PC-graph of the first three bars of Schoenberg's *Klavierstück* op.33a (Example 4.36)[318] illustrates this. Its a curve that never exceeds a degree of prime consonance of 80 (the highest value being $PC_{(3\text{-}7)} = 78$, and goes down as low as $PC_{(5\text{-}4)}$ and $PC_{(7\text{-}10)} = 19$. $PC_{(av)} = 40{,}92$.



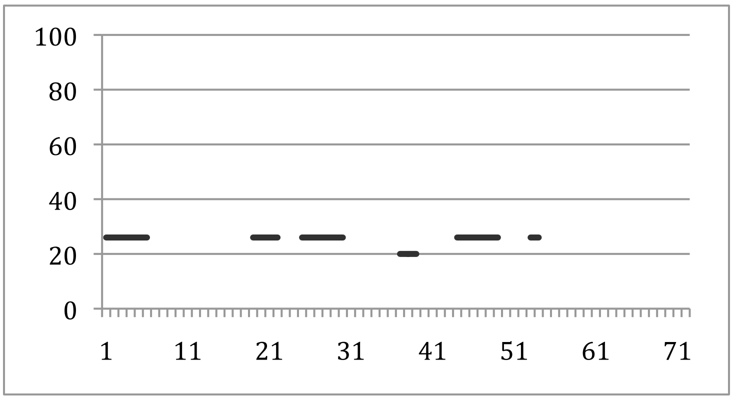
Example 4.36: PC-graph of Schoenberg's *Klavierstück* op.33a, bar 1-3
(unit of the horizontal axis = quaver).

The PC-analysis of my piano piece *Après la pluie* (see T-analysis in section 3.5.5) reveals a PC-curve that is at a constant low degree of prime consonance, as can be seen in Example 4.37. However, this graph is exceptional. The 'flat' shape of the curve (disregarding the gaps caused by silence or single pitch classes) is the result of the fact that the three analysed bars from *Après la pluie* are based (apart from the many single pitch classes) on a very restricted number of different set classes with a

[317] Arnold Schoenberg, *Style and idea*, Leonard Stein, (ed.), Leo Black (transl.). Berkeley & Los Angeles: University of California Press, 1975, p. 260.

[318] Note how the symmetric shape of the first part of the curve reflects the symmetrical structure of the first two bars of the piece.

comparable (very low) degree of prime consonance: [2-1], [3-1] and [4-1], all representing chromatic pitch classes with no 'holes' (strict clusters or their inversions)[319].



Example 4.37: PC-graph of *Après la pluie* for piano, bar 1-3
(unit of the horizontal axis = demi-semiquaver).

One might conclude from this graph that a 'constant' low degree of prime consonance is represented by a 'flat' curve, with no big fluctuations. This, however, is not true. Greater variation in the set classes used inevitably results in a PC-graph with greater fluctuations; in that case, the degree of consonance is not 'constant'—the PC-curve is no longer flat—but can still be 'constantly low' as is shown in the analysis of the first three bars of my piano piece *A l'image du monde…originel* (2013) shown in Example 4.38.

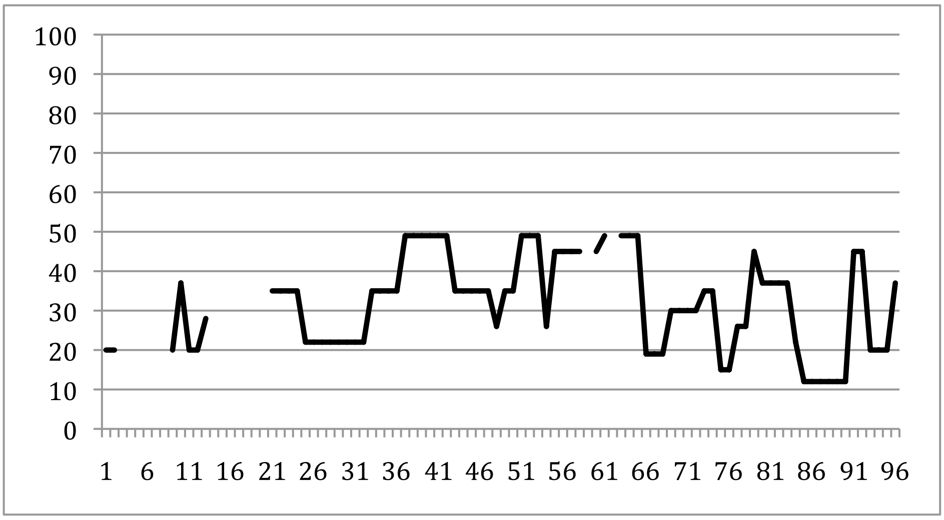
Example 4.38: *A l'image du monde…originel,* for piano (2013) bar 1-3.

The PC-graph of this excerpt is more typical for highly dissonant music, the PC-values of highly dissonant music are variable but they never (or rarely) exceed the symbolic threshold of 50.[320]

A 'constant' degree of prime consonance is represented by a PC-curve with a restricted range, regardless of the absolute PC-values. A piece has a constant high degree of prime consonance if its degree of prime consonance fluctuates within a restricted range of high PC-values; it has a constant low degree of prime consonance when it has a restricted range of low PC-values. The PC-graph of *A l'image du monde…originel*, represented in Example 4.39 shows a constantly low degree of

[319] That is, if ic 1 can be called a cluster (consisting of two pitch classes).
[320] The maximum PC-value of CIG's is kept under 50 in the construction of the PC-formula in order to create this symbolic threshold.

consonance: it never exceeds a PC-value of 50 and the piece is therefore highly dissonant. It is also constantly low because its extreme PC-values (12 and 49) stay within a restricted range of 37.



Example 4.39: PC-graph of *A l'image du monde…originel* for piano, bar 1-3
(unit of the horizontal axis = demi-semiquaver).

We can compare this value with the ranges of the other analysed pieces. Bach's *Aria* has PC-values between 100 and 26; it has a very broad PC-range of 74 (100 – 26). Even if we ignore the brief [2-1] (with $PC_{(2\text{-}1)}$ = 26), which may be said to distort the picture, the PC-range still amounts to a high 59, comparable to the PC-ranges of the *Tristan* prelude (55) and Schoenberg's op.33a (59). Bach's *25th Goldberg Variation* on the other hand evolves within a more restricted PC-range of 40 (100-60); it is therefore said to have a constant high degree of prime consonance.

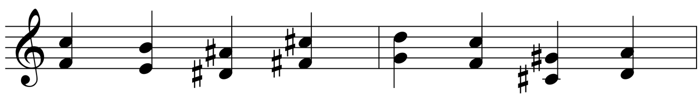
Note that *A l'image du monde…originel* features an abundant use of sustain pedal. This is ignored in the PC-analysis. The actual perceived degree of prime consonance of the piece is certainly lower than the analysis indicates, since more complex chords are heard. However, the effect of the sustained sounds is hard to represent in the analysis because of the decay in intensity of the piano sound (the tones are not strictly sustained), and there is an undeniable difference in perceived sensory consonance between attacked sound complexes and sounds combinations with tones added to previous sounds. The complexity of this phenomenon makes it impossible to take pedal use into account in PC-analysis. However, this does not change much to the eventual result: even when the pedal is ignored, the degree of prime consonance of *A l'image du monde…originel* is constantly low.

Conversely, in the PC-analysis of the Bach pieces, grace notes were interpreted as sounding at the same time as the notes they are attached to. This lowered the degree of prime consonance of the pieces. Still, the graphs show that Bach's music is highly consonant, and that the high degree of prime consonance of the *25th Variation* is constant. The conclusions of PC-analysis would be even more outspoken if applied more 'strictly'.

4.6.2 Relation between tonality and consonance

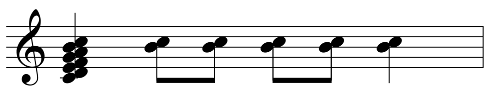
Tonality and consonance are often understood as correlated phenomena. It is indeed true that much highly atonal music is at the same time highly dissonant, and that most music belonging to common-practice is highly consonant. However, an increase in chromaticism (resulting in a lower degree of tonality) does not always entail a decreasing degree of prime consonance. This is obviously the case for monophonic music, since the concept of consonance doesn't apply to it. But it is also true for polyphonic music.

We saw, for instance, that the average degrees of prime consonance of the highly diatonic *Aria* from Bach's *Goldberg Variations* and the highly chromatic (and therefore less tonal) *25th Variation* were almost identical. There is no prerequisite correlation between diatonicity/chromatism or degree of tonality on the one hand and degree of prime consonance on the other. Highly chromatic music usually has a lower (average) degree of tonality, but not necessarily a lower degree of prime consonance. Highly chromatic music can be highly consonant, as the Wagner analysis showed. The short phrase in Example 4.40 is an extreme example. This phrase has an average degree of prime consonance $PC_{(av)}$ = 100, since it consists of harmonic intervals of perfect fifths throughout,. Still, it is highly chromatic and therefore it has a relatively low average degree of tonality $T_{(av)}$ = 33,25.[321]

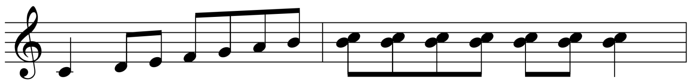


Example 4.40: Highly chromatic phrase with $PC_{(av)}$ = 100.

Conversely, a highly diatonic and consonant piece can have a very low average degree of prime consonance, as is illustrated by the (extreme) cases of Example 4.41 a & b. The first phrase (Example 4.41 a) has an immediate and constant degree of tonality of $T_{(7\text{-}35)}$ = 100, but a low average degree of prime consonance $PC_{(av)}$ = 33,50. The phrase in Example 4.41 b has a degree of consonance that gradually builds up to $T_{(7\text{-}35)}$ = 100 during the first bar. It also has a relatively high average degree of tonality[322]: $T_{(av)}$ = 67, but a degree of prime consonance that is even lower than that of the first phrase: it stays constantly at $PC_{(2\text{-}1)}$ = 26, since the only harmonic intervals in this phrase are minor seconds.



a



b

Example 4.41 a & b: Highly diatonic and highly tonal phrases
with a low average degree of prime consonance.

Although there is no necessary correlation between tonality and consonance, there is a remarkable feature of the degree of prime consonance of the tonal 7-sets ([7-32], [7-34] and [7-35]). In his 1994 study on sensory consonance mentioned before, David Huron notices: "Among the pitch-class sets whose interval-class inventories conform most strongly with an index of perceived consonance are the three preeminent scales in Western music: the major diatonic scale, and the harmonic and melodic minor scales"[323]. Allan Smith rephrases Huron's findings, stating: "in sets of 7 tones, 21 intervals

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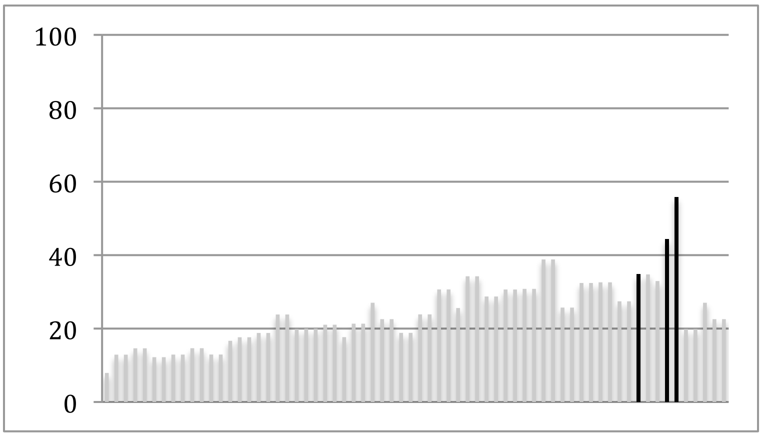
[321] The degree of tonality evolves per crotchet beat as follows: 14, 14, 65, 52 (maximum degree of tonality of the phrase), 38, 37, 32.

[322] This is the evolution of the degree of tonality of the phrase: 1 during a crotchet, then quavers at 13, 19, 26, 58, 87, and finally 100 for the last quaver of the first bar and the entire second bar.

[323] David Huron, *Interval-Class Content in Equally Tempered Pitch-Class sets*: *Common scales exhibit optimum tonal consonance*, in *Music Perception: an Interdisiplinary Journal*, Vol. 11, N°3, 1994, p. 303.

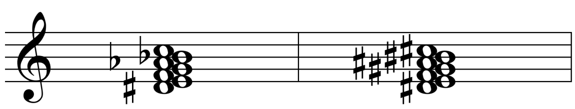
existed among the tones. The most consonant combinations of these intervals were found in 7 sets, corresponding to the major diatonic scale, the natural minor scale, and the traditional modes"[324]

This same conclusion can be drawn from the values obtained with the PC-formula. All 7-sets (pitch class sets or set classes with cardinality 7) have a low degree of prime consonance, merely due to the fact that they contain 7 pitch classes, so more dissonant intervals between their constituting pitch classes are unavoidable. But if we look at the PC-values for all 7-sets (shown in Example 4.42), we notice that the tonal 7-sets (the darker lines in Example 4.42) are indeed amongst the most consonant within the cardinality group: $PC_{(7\text{-}32)} = 35$ and $PC_{(7\text{-}34)} = 44$; the value for [7-35], $PC_{(7.35)} = 56$, can even be termed 'high'.



Example 4.42: PC-values for 7-sets.
[7-32], [7-34] and [7-35] are indicated darker.

The only set classes with a degree of prime consonance in the same range ($PC_{(7\text{-}n)}$ between 35 and 56) are (apart from the obvious [7-32i]): [7-27] and [7-27i]. Example 4.43 a & b show an instance of both set classes. The high degree of prime consonance of both set classes is caused by the fact that their interval class vector <344451> contains at the same time a low number of ic 1's (only 3, 2 being the lowest possible number) and a high number of ic 5's (5, 6 being the highest possible number). Not surprisingly, set classes [7-27] and [7-27i] are very similar to a tonal 7-set: replacing D sharp by D natural in Example 4.43 a, and F natural by F sharp in Example 4.43 b, turns the former into the set of f melodic minor and the latter into c sharp melodic minor, both instances of [7-34].



a b

Example 4.43: Examples of pc-sets
belonging to set classes [7-27] (a) and [7-27i] (b).

In summary, although the tonal 7-sets have a relatively high degree of prime consonance within their cardinality group, and (by definition) the highest degree of tonality, this correlation is not necessarily reflected in musical implementation. Highly tonal music may have a low degree of prime consonance, while highly atonal music may possess a high degree of consonance. This certain degree of independence is caused by the fact that tonality is a context-related feature, whereas sensory consonance is not.

[324] Allan B. Smith, *A "Cumulative" Method of quantifying Tonal Consonance in Musical Key Contexts"* in Music Perception: An Interdisciplinary Journal, Vol. 15, N°2 (Winter, 1997), University of California Press, pp. 179.

With musical (context related) consonance, the picture would probably be different. Although musical consonance doesn't lie within the scope of the present research, I want to briefly address it in the light of the tonality-consonance relation. As was discussed before, there are two kinds of musical consonance: 1) relative musical consonance (consonance determined by the contrast between successive pc-sets of different degrees of consonance), and 2) resolution-based or functional musical consonance.

Relative musical consonance of a chord in a piece could be determined by calculating the 'retrograde' average degree $PC_{(R)}$ of prime consonance of the preceding sounds in the piece. That is the average degree of prime consonance of the section of the piece immediately preceding the chord where all the PC- values lies within a determined narrow range around the average. The relative musical consonance of the chord in the piece is then determined by the deviation of its degree of prime consonance from $PC_{(M)}$ multiplied by a factor that depends on the length of the phrase over which $PC_{(M)}$ is calculated. The multiplying factor is necessary because contrasts in perceived relative musical consonance get stronger when the contrasting preceding average degree of prime consonance lasts longer. This gives a value Δ (delta) for relative musical consonance that can be calculated with a formula of the following form:

$$\Delta_{(x)} = L_{(R)} \,.\, [PC_{(x)} - PC_{(R)}]$$

where $\Delta_{(x)}$ is the relative musical consonance of chord x, and $PC_{(x)}$ is the degree of prime consonance of x. $L_{(R)}$ is the factor determined by the length L of the phrase immediately preceding x with PC-values within the predetermined narrow range around $PC_{(R)}$. If this value is positive, x is relatively more consonant than the preceding music over length L; if it is negative, it is relatively more dissonant. The precise determination of factor $L_{(R)}$ exceeds the scope of the present research.

Resolution-based musical consonance is less easily quantifiable. It may depend on the number of common pc's between two adjacent pc-sets, but a further development of this idea does not lie within the scope of the present research, which only focuses on sensory consonance, since musical consonance plays only a minor role in CIG-serialism the way I use it. There are two reasons why musical consonance is only of marginal importance in my technique and idiom: first of all, CIG-serialism pursues constant low degrees of consonance, and therefore large contrasts in degree of prime consonance are limited. The C-curve is kept as 'flat' as possible, i.e. within a limited range. Therefore also the values for $\Delta_{(x)}$ will stay relatively low. Secondly, by using rhythmic cells that extend the occurrence of each series note in time, there is always a high number of common pc's between adjacent pc-sets. This results in music without contrasts in 'functional' tension worth mentioning, and therefore no fluctuations in resolution-based consonance worth mentioning..

To bring this chapter to a close, a last remark is appropriate. As was mentioned before, 'other aspects' that influence the perception of consonance, such as timbre or loudness are not taken into account in PC-analysis. Only pitch classes with the same or 'similar' timbre, loudness, etc. are compared, regardless their timbre or loudness. Changing those parameters would change the absolute values of degrees of prime consonance, but not necessarily their relative values within the same timbre or loudness ranges. A very loud or very soft minor second will still be more dissonant than a perfect fifth of the same loudness (within acceptable limits)[325]. Since my aesthetic goal is not only to work with sound compounds of comparable degrees of consonance (low PC-ranges and low values of $\Delta_{(x)}$), but also to obtain homogeneously sounding music, large instantaneous contrasts of timbre and loudness hardly ever occur, and therefore significant contrasts in perceived consonance due to timbre of loudness are mostly absent from my pieces. Timbre, loudness and other parameters usually evolve in a gradual manner in my music, albeit often with extreme ranges (this is certainly the case with the dynamic range of most of my music).

[325] It may be the case that differences in degree of consonance become hard or impossible to discern for sounds at the auditory thresholds (absolute threshold of hearing (ATH) and threshold of pain).

4.7 Additional remarks

It is important to note that the T-formula and PC-formula developed in the present research are not the only ones in their kind; they may not even be final versions. As was discussed, other researchers have developed theories and formulas to quantify aspects of tonality or consonance of tone combinations that are more or less similar to the formulas developed in chapters 3 and 4. However, none of them turn out to fit the present context adequately. The T and PC-formulas are different from the e.g. Robert Morris's 1979 research on "similarity indices"[326], from Isaacson's "Interval-class Vector Similarity-relation"[327], which are "based on the interval-class-vector (V) associated with the sets that are members of a SC [set class]"[328], or on "interval content of pitch-class set classes",[329] and from "Forte's Rn relations [that] indicate the degree of correspondence between the respective terms of the [interval class vectors] of a pair of sets of the same cardinality"[330]. Also Dmitri Tymoczko's idea of "musical distance"[331], or Ian Quinn's "Unified Theory of Chord Quality"[332], mentioned before, do not fit the present purpose adequately. Amongst the reasons why the other theories don't fit the present context can be sited: 1) the fact that similarity indices compare two pc-sets, whereas the T-formula is the result of the comparison of 4 pc-sets (the test set with [7-32], [7-34] and [7-35]), and 2) similarity indices based on interval content in interval-class vectors result in identical indices for Z-related pc-sets and their inversions, which is not necessarily true in the case degrees of tonality determined with the T-formula. The T- and PC-formulas reflect my specific path of reasoning more precisely. As will be discussed in Part 2and 3, they describe the endophysical laws of my personal aesthetic universe.

Finally, note that the degree of tonality of a series can be determined, but not its degree of prime consonance. The concept of prime consonance of a pitch class set relates only to simultaneously sounding tones. A series consists of consecutive notes. Only the simultaneous use of these notes results in a certain degree of prime consonance. A CIG-series therefore has no degree of prime consonance, but allows for the composition of music with a constant very low degree of prime consonance.

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326 Robert Morris, *A Similarity Index for Pitch-Class Sets*, in Perspectives of New Music, Vol. 18, N°1/2, 1979, pp. 445-460.
327 Eric J. Isaacson, *Similarity of Interval-Class Content between Pitch-Class Sets: The IcVSIM Relation*, in Journal of Music Theory, Vol. 34, N°1, 1990, pp. 1-28.
328 Robert Morris, *A Similarity Index for Pitch-Class Sets*, in Perspectives of New Music, Vol. 18, N°1/2, 1979, p. 446.
329 Eric J. Isaacson, *Similarity of Interval-Class Content between Pitch-Class Sets: The IcVSIM Relation*, in Journal of Music Theory, Vol. 34, N°1, 1990, p. 2.
330 Eric J. Isaacson, *Similarity of Interval-Class Content between Pitch-Class Sets: The IcVSIM Relation*, in Journal of Music Theory, Vol. 34, N°1, 1990, p. 2.
331 Dmitri Tymoczko, *Three Conceptions of Musical Distance*, in: Elaine Chew, Adrian Childs, & Ching-Hua Chuan (eds.), *Mathematics and Computation in Music*, Springer, 2009, pp. 258-273; Dmitri Tymoczko, *Geometrical Methods in Recent Music Theory*, Music Theory Online 16.1, 2010, http www.mtosmt.org/issues/mto.10.16.1/ mto.10.16.1.tymoczko.html [last accessed: 24 January 2013].
332 Ian Quinn, *A Unified Theory of Chord Quality in Equal Temperaments*, unpublished doctoral dissertation, 2004.

Chapter 5. Assessment and adaptation of the CIG-technique

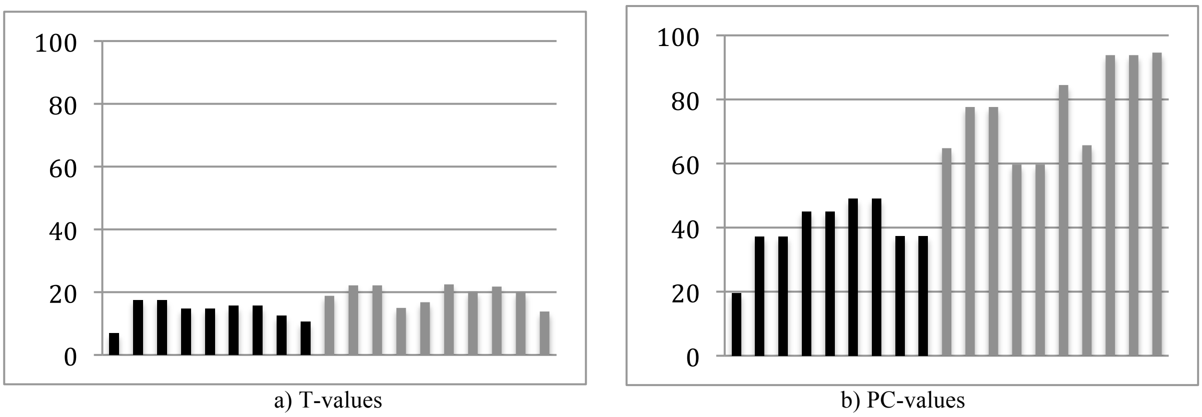
5.1 Introduction

Now that degrees of tonality and prime consonance have been determined, the initial claim that CIG-serialism yields highly atonal and dissonant music can be assessed, and the final research question — whether and how the technique of CIG-serialism can be adapted in order to result in even lower degrees of tonality and consonance—can be answered. This is the purpose of the present chapter.

5.2 Assessment of CIG-serialism

When the graphs representing the T- and PC-values of set classes calculated with the T- and PC-formulas are compared, it is obvious that both graphs have a different shape. Tonality values tend to increase from cardinality 1 to 7, and decrease again towards the highest cardinality groups (see Examples 3.27 and the list of all degrees of tonality in Appendix 1). Prime consonance values are the highest for cardinality group 1, and decrease gradually with increasing cardinality (see Examples 4.30 and the list of all degrees of prime consonance in Appendix 1). This indicates that tonality and consonance are independent aspects of music to a certain degree. Still it was noticed that there is some correlation between the two. Indeed, T-values and PC-values show a remarkable similarity reflected in their graphs. This similarity occurs within groups of set classes with the same cardinality.

Example 5.1 shows the graphs for T-values and PC-values of all set classes of cardinality 3. The graphs can be divided in two parts. The group containing the first nine set classes (indicated in black in Example 5.1) has T-values and (more outspoken) PC-values that are generally lower than the second part of the cardinality group.



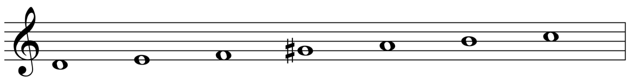
Example 5.1: T-values (a) and PC-values (b) for cardinality group 3 compared.
The group with lower values is indicated in black.

The set class group with the lower degrees of tonality and prime consonance is the one that contains the CIG's used in Smith Brindle's 'atonal series' and in CIG-serialism. This seems to confirm Reginald Smith Brindle's claim that the use of CIG's ("note-groups of a chromatic nature"[333]) results in 'atonal series' (series which maintain throughout the same degree of atonality), and seems to

[333] Reginald Smith Brindle, *Serial Composition*, Oxford University Press, 1966, p. 12.

provide the answer to the central research question on the present text that CIG-serialism indeed results in highly atonal and dissonant music.

This conclusion is premature however, because, although the use of CIG's in the construction of series—and in the composition with such series— increases the probability that the resulting piece will have a low degree of tonality and prime consonance, this is not necessarily the case. The low degrees of tonality and prime consonance on a three note level may disappear on a broader level. For example, a harmonic minor scale ([7-32]) could be arranged in such a way that it consists entirely of CIG-3's and could be part of a CIG-series, as is shown in Example 5.2:



Example 5.2: Harmonic minor scale as part of a CIG-series.

The degree of tonality of every CIG-3 contained in this example is low, but the degree of tonality of the whole is $T_{(7\text{-}32)}$ = 100. When the series in Example 5.2 occurs in an alleged 'atonal series', the degree of tonality of the series in Example 5.2 rises after the first three notes as soon as the fourth pitch class is added to it, in other words, when the CIG is extended into a pitch class set of cardinality 4. With every additional extension of the series, the degree of tonality increases, as can be seen in the table of Example 5.3. Also, the degree of prime consonance stays relatively constant (between 34 and 40) when the series is extended, whereas it should go down significantly with increasing cardinality if a low degree of prime consonance is envisaged. Note that the scale in Example 5.2 may be part of one of Reginald Smith Brindle's atonal series, but not of a CIG-series, since the same CIG (ascending ic 2 followed by ascending ic 1) occurs twice (at the very beginning and the very end of the scale), which is forbidden in CIG-series. But the sequence of the first six pitch classes is possible in a CIG-series, which would still raise the degree of tonality to an unacceptably high $T_{(6\text{-}z29)}$ = 73.

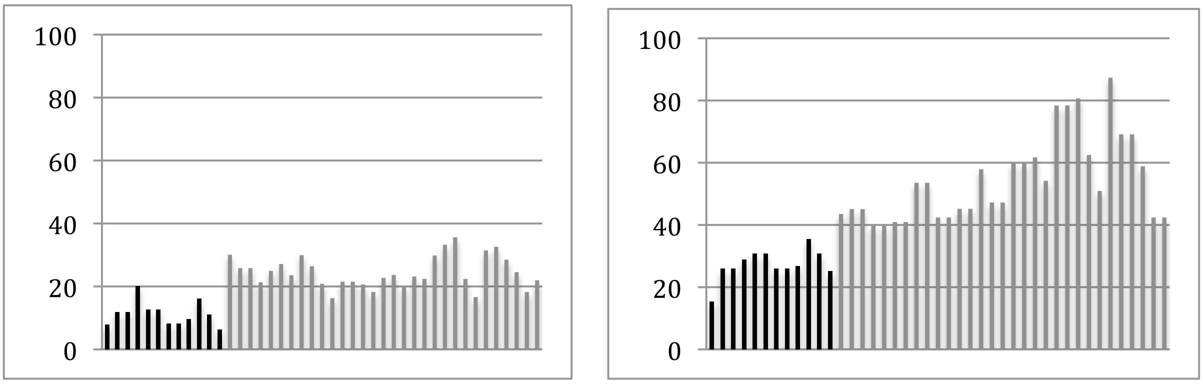
set class name	$T_{(c\text{-}n)}$	$PC_{(c\text{-}n)}$
3-2	18	37
4-12	21	40
5-z18i	34	34
6-z29	73	36
7-32	100	35

Example 5.3: T- and PC-value evolution
in the construction of the series of Example 5.2.

Combinations like the sequence of the first six pitch classes in Example 5.2 should be forbidden if a high degree of atonality and dissonance is to be persued. Reginald Smith Brindle's claim that series consisting exclusively of CIG's have a (constant) high degree of atonality proves to be insufficient, even with the restriction imposed by the CIG-technique in the construction of series. It is therefore not enough to construct a series with CIG's; the groups of higher cardinality need to have a low degree of tonality and prime consonance as well. Therefore the CIG-technique needs to be adapted and an extra restriction has to be introduced.

5.3 CIG's of higher order

The phenomenon of the group of CIG's with lower degrees of tonality and prime consonance within the set class group of cardinality 3 (see Example 5.1) also occurs within the set class groups of higher cardinality. The contrast between the set classes with low T- and PC-values and the others gets even more outspoken with increasing cardinality, as can be seen in Example 5.4, where T-values and PC-values for cardinality group 4 are shown. In this cardinality group the degrees of tonality and prime consonance of the first twelve set classes as a whole (indicated in black in the graph) is clearly lower than that of the rest of the cardinality group.



a) T-values b) PC-values

Example 5.4: T-values (a) and PC-values (b) for cardinality group 4 compared.
The group with lower values is indicated in black.

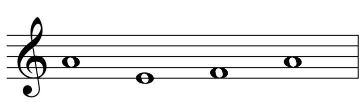
A closer assessment of the first twelve set classes in the group of cardinality 4 (with extended forte names from [4-1] through [4-9]) reveals a remarkable feature shared by their prime forms (and inverted forms): the difference between 2 consecutive pitch class numbers in the prime forms of these set classes equals 1 in at least two of the three cases. For example, the prime form of set class [4-2] is (0,1,2,4). The difference between two successive pitch class numbers in this string is 1 between pitch classes with pc-numbers 0 and 1 (the first two pitch classes in the ordered set) and between pitch classes with pc-numbers 1 and 2, but not between pitch classes with pc-numbers 2 and 4 (the last two in the ordered set) that are ic 2 apart. A non-chromatic interval between two notes in the prime form of a pitch class set or set class will henceforth be called the **non-chromatic gap**.

This means that, when the pitch classes in the prime form or inverted form of a pitch class set belonging to one of the twelve set classes of cardinality 4 in the low T- and PC-group are placed in ascending (or descending) order, there is never more than one non-chromatic gap, one interval between two successive notes that is not an (instance of) ic 1. Example 5.5 shows the prime forms (and inverted forms) of the twelve set classes thus ordered. This leads to an extension of the concepts of chromatic pc-set and CIG. Remember that chromatic pitch class sets of order 3 were defined as 'pc-sets containing 3 pitch classes, at least two of which are ic 1 apart'[334]; this is to say that their prime form has one non-chromatic gap at most. CIG's were defined as 'ordered pc-sets derived from chromatic pc-sets of order 3'[335]. Likewise, pc-sets of cardinality four with not more than one non-chromatic gap (in their prime form) are called **chromatic pc-sets of order 4**, and ordered pc-sets derived from chromatic pc-sets of order 4 are defined as **chromatic interval groups of order 4** or **CIG-4**'s.

[334] See Section 1.1.2.
[335] See Section 1.1.3.

Example 5.5: Prime forms of all CIG-4's.
The single non-chromatic gap is indicated with a horizontal bracket.

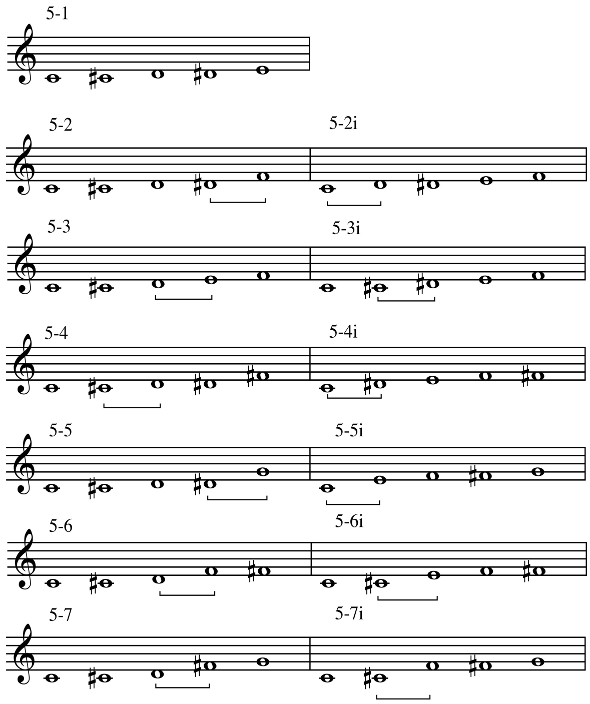
The term CIG is thus no longer restricted to sets with cardinality 3. What was previously called a CIG (with no further specification of cardinality) will therefore henceforth be called a **CIG-3** to distinguish between the CIG's of different cardinalities. Note that four-note groups within a series may consist of only three different pitch classes; these are henceforth also called CIG-3's with specification of the number of notes in the group. Example 5.6 shows a **4-note CIG-3**.[336] The first three notes (A-E-F) constitute a CIG-3. The next note (A) is a repetition of the first note, so the 4-note group is still a trichord.



Example 5.6: a 4-note CIG-3.

In general a **CIG of cardinality n**, or **CIG-n**, is defined as an ordered pitch class set consisting of n different[337] pitch classes, at least n-1 of which are ic 1 apart (for n = 3 to 12) in their prime form, or as ordered pc-sets whose prime form contain one non-chromatic gap at most. Example 5.7 shows the prime form of all CIG-5's.

[336] In the analyses of 56-CIG pieces in part 4, another special case will be discussed: chromatic 3-note groups with only two different pitch classes. These are actually 'CIG-2's' (although CIG's are only defined for groups of cardinality 3 and higher) since they are based on ic 1 or set class [2-1], and are therefore groups with only two different elements.
[337] The specification 'different' allows for repeated pitch classes within CIG's.



Example 5.7: Prime form of all CIG-5's.
The single non-chromatic gap is indicated with a horizontal bracket.

The number of ic 1's (the first number) in the interval class vectors of any CIG-n is n-1 for instances of set classes with forte number [n-1], and n-2 for all other CIG-n's.[338] The ic-vectors of all other set classes have a number of ic 1's lower than n-2. This rule will be called the **interval class vector rule for CIG's**, or short **ICV-rule** in further arguments and proofs.

For example, CIG-4's have ic-vector <321000> for instances of [4-1], and an ic-vector of the form <2…..> for all other CIG-4's . All other set classes of cardinality 4 have an ic-vector of the form <1…..> or <0…..> (the five dots in the ic-vectors have to be replaced by the number for ic's 2 to 5).

Example 5.8 lists the set classes of the CIG's of all cardinalities with their degrees of tonality and prime consonance. The cardinality groups of CIG's correspond to the groups with 'low' T- and PC-values for all cardinalities higher than 2 discussed before. Note that the CIG-6's do not occur as a 'group' in the graphs of Examples 3.27 and 4.30. Note also that all set classes with cardinalities higher than 9 represent CIG's.

As can be seen in Example 5.8, the degree of tonality of CIG's (of any cardinality) never exceeds 32 (the T-value for [6-z3i]); their degree of prime consonance is never higher than 49 (for [3-4] and [3-4i]). It is therefore justified to say that music that is based exclusively on the use of CIG's of all cardinalities yields the most atonal and dissonant result. Starting from this finding, the construction rules for CIG-series can be adapted in order to obtain the lowest possible degrees of tonality and prime consonance.

[338] This characteristic has a formal mathematical proof, but it can also be deduced after listing the ic-vector of all set classes (see Appendix 2).

set class name	$T_{(c-n)}$	$PC_{(c-n)}$
3-1	7	20
3-2	18	37
3-2i	18	37
3-3	15	45
3-3i	15	45
3-4	16	49
3-4i	16	49
3-5	12	37
3-5i	11	37
4-1	8	15
4-2	12	26
4-2i	12	26
4-3	20	29
4-4	13	31
4-4i	13	31
4-5	8	26
4-5i	8	26
4-6	10	27
4-7	16	36
4-8	11	31
4-9	6	25
5-1	6	12
5-2	19	20
5-2i	19	20
5-3	18	22
5-3i	18	22
5-4	14	19
5-4i	14	19
5-5	13	20
5-5i	13	20
5-6	8	22
5-6i	10	22
5-7	5	20
5-7i	5	20
6-1	15	10
6-2	26	14
6-2i	22	14
6-z3	28	16
6-z3i	32	16
6-z4	16	16
6-5	18	16
6-5i	18	16
6-z6	12	17
6-7	4	14
6-z36	21	16
6-z36i	24	16
6-z37	22	16
6-z38	19	17

set class name	$T_{(c-n)}$	$PC_{(c-n)}$
7-1	4	8
7-2	22	13
7-2i	22	13
7-3	19	15
7-3i	20	15
7-4	15	12
7-4i	15	12
7-5	13	13
7-5i	13	13
7-6	9	15
7-6i	16	15
7-7	4	13
7-7i	4	13
8-1	3	6
8-2	21	11
8-2i	21	11
8-3	20	12
8-4	24	13
8-4i	24	13
8-5	10	11
8-5i	10	11
8-6	17	11
8-7	17	15
8-8	9	13
8-9	8	10
9-1	2	4
9-2	20	9
9-2i	20	9
9-3	22	12
9-3i	22	12
9-4	16	13
9-4i	16	13
9-5	16	9
9-5i	12	9
10-1	2	3
10-2	23	9
10-3	29	13
10-4	20	15
10-5	26	16
10-6	15	8
11-1	1	2
12-1	1	1

Example 5.8: List of chromatic pc-sets for all cardinalities
with their T- and PC-values.

5.4 Adaptation of CIG-serialism: general CIG-serialism

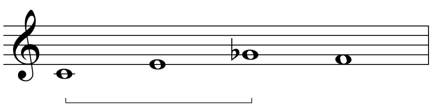
5.4.1 Extending CIG-3's

To obtain the highest possible degrees of atonality and dissonance—which is the aim of the present research—CIG's should consist exclusively of CIG's of all cardinalities. Whenever a succession of any number of pitch classes higher than 3 in a CIG-3 series is not an ordered chromatic pitch class set, its degree of consonance and/or prime consonance may rise, as is the case in the harmonic minor group in Example 5.2. Indeed, although all groups of three successive notes in this example are CIG's, the groups of more than three successive notes are not always CIG's. The ordered pitch class set formed by the first four pitch classes (D-E-F-G sharp) in example 5.2, for instance, is an instance of [4-12]. Notes 2 to 5 (E-F-G sharp-A) are an instance of [4-7]—and therefore a CIG-4—but notes 3 to 6 (F-G sharp-A-B) are again an instance of [4-12]. None of the groups of cardinalities 5, 6, and 7, in the example are CIG's. Successions such as the first or last six pitch classes in Example 5.2—which are theoretically possible in the original CIG-serialism—should be ruled out in the construction of CIG-series if the aim is to obtain the highest possible degrees of atonality and dissonance.

The question is then how to construct CIG-series that consist entirely of CIG's of all cardinalities. In the first place, the initial construction rules[339] remain intact:

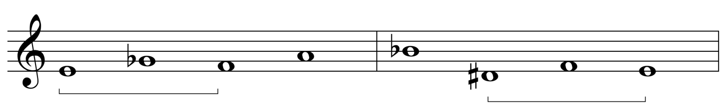
- In a CIG-series, all three successive pitch classes are CIG-3's.
- Each of the 54 CIG-3's occurs exactly once.
- A CIG-series is closed (this way amotivity is guaranteed).

Of these rules, the second cannot be extended to CIG's of higher order. It is impossible to construct a CIG-series with all CIG-3's and all CIG-4's occurring exactly once simultaneously. Experiments with CIG-4's made it immediately clear that constructing series based on CIG-4's is problematic; not only is the number of possible CIG-4's too high for practical use, an additional problem is that some CIG-4's do not consist of two interwoven CIG-3's—which disqualifies them from the outset, since it breaks construction rule 1—and CIG-3's occur more than once in all possible CIG-4 combinations, which is a breach of construction rule 2. An example of a CIG-4 that does not consist of 2 CIG-3's is shown in Example 5.9. The last three notes in the CIG-4 of this example form a CIG-3 (instance of [3-1]), but the first three notes don't (they are an instance of [3-8i]), and therefore the CIG-4 cannot be used in the construction of the CIG-series. Example 5.10 shows two different CIG-4's that cannot occur simultaneously in a CIG-series, because they contain the same CIG-3 (only distinct by transposition), which would make the series motivic.



not a CIG-3

Example 5.9: A CIG-4 not consisting of two CIG-3's.
The first three notes (indicated with square brackets)
constitute no CIG-3 (it is an instance of [3-8i]).



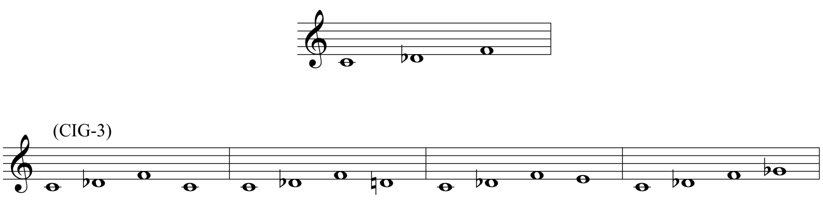
Example 5.10: Two different CIG-4's containing the same CIG-3
(indicated with square brackets. The common CIG-3
is transposed down a semitone in the second CIG-4).

[339] See Section 1.1.4.

The objections to the use of CIG-4's as a basis for the construction of CIG-series only intensify for CIG's of higher cardinality. Therefore, CIG-3's as the basis for CIG-serialism remains the best option, and the only practically feasible. The aim is now to order the CIG's in such a manner that they produce series in which all groups of higher cardinality are CIG's. Starting from an arbitrary CIG-3, a series will be constructed by cumulatively adding new notes that are the last note of a CIG-3 that did not occur before. Each new note also has to be part exclusively of CIG's of all cardinalities up to the cardinality of the complete series up to that point.

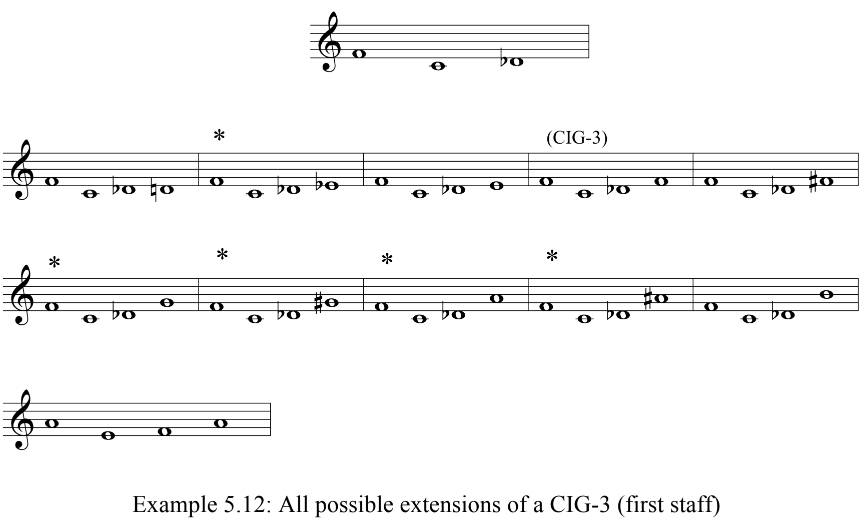
5.4.2 From CIG-3 to CIG-4

In a first step, a CIG-3 is extended to form an ordered pitch class set with four notes consisting of two interwoven CIG-3's; the ordered four-note set is either a tetrachord or a 4-note CIG-3 (when the first and last note are instances of the same pitch class). In most cases the tetrachord will automatically be a CIG-4. When, for instance, the CIG-3 in Example 5.11 is extended with one additional note in such a manner that the last three notes form a (different) CIG-3, the resulting four-note group is automatically a CIG-4 (or a 4-note CIG-3, as in the first case). All other possible extensions are excluded because they would not result in a CIG-3 for the last three-note group.



Example 5.11: All possible extensions of a CIG-3 (first staff)
automatically resulting in CIG-4's (or a 4-note CIG-3 as in the first case)
consisting of 2 CIG-3's.

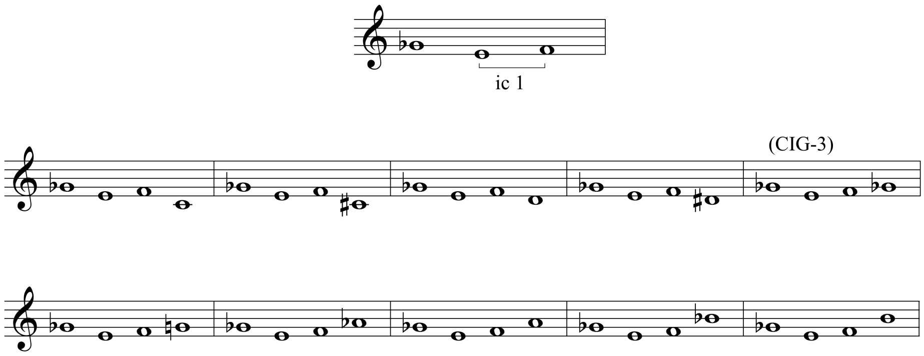
Tetrachord extensions of CIG-3's do not always result in CIG-4's, even if the additional note forms a CIG-3 in combination with the previous two notes. In Example 5.12, for instance, there are 11 ways to extend the CIG-3 consisting of F, C and D flat in such a manner that the added note forms a CIG-3 with the preceding C and D-flat. Five of these extensions result in a tetrachord that is not a CIG-4. These are indicated with an asterisk (*) in Example 5.12. One extension results in a 4-note CIG-3 (indicated with "CIG-3" in Example 5.12).



Example 5.12: All possible extensions of a CIG-3 (first staff)
not exclusively resulting in CIG-4's (non-CIG-4's are indicated with *).
One group is a 4-note CIG-3.

The significant difference between Examples 5.11 and 5.12 is the position of the chromatic interval class (ic 1) in the initial CIG-3. When the initial CIG doesn't end on a chromatic interval (ic 1), an extension of the CIG-3 with an additional note that is the last note of a new CIG-3 always results in a CIG-4 or a 4-note CIG-3. Indeed, when the last two notes of the initial CIG-3 are not ic 1 apart, they can only become the first two notes of a CIG-3 if the additional pitch class is at ic 1 distance from at least one of those two notes, otherwise a second non-chromatic gap is created and the final three notes do not constitute a CIG-3. In Example 5.11, D flat and F are not ic 1 apart. They can only become the first two notes of a new CIG-3 if the additional note is C, D, E or G flat (ic 1 distance from D flat or F). The four-note group thus obtained is always a CIG-4 or 3-note CIG-3. There can never be an extra non-chromatic gap in the prime form of the four-note group (see Example 5.5); therefore extensions of CIG-3 that do not end on ic 1 are automatically extended into CIG-4's or 4-note CIG-3's.

Extensions that do not result in either 4-note CIG-3's or CIG-4's can only occur when the initial CIG-3 ends on a chromatic interval (ic 1), as is the case in Example 5.12 (between C and D flat). Indeed, there are more possibilities to turn the group C-D flat into a CIG-3 by adding a note; any other pitch class can be added. In some cases the resulting tetrachord is not a CIG-4. If the added note is not at ic 1 distance from the two previous notes it has to be at ic 1 distance from the first note, otherwise an additional non-chromatic gap is created. The only exception occurs when the initial CIG-3 is a permutation of set class [3-1] (which has no non-chromatic gap). In that case all extensions result in CIG-4's (or a 4-note CIG-3 in one case) as can been seen in Example 5.13. Indeed, since there was no non-chromatic gap in the initial CIG-3, there can be one non-chromatic gap at most after extension.



Example 5.13: Extension of a CIG-3 that is an instance of set class [3-1] (first staff) resulting in nine possible CIG-4's and one 4-note CIG-3.

The assertion that extensions of CIG-3's (with one note that forms a CIG-3 with the previous two notes) always result in either 4-note CIG-3's or CIG-4's except in some cases where the initial CIG-3 ends with the chromatic interval (ic 1) is called the **extension rule for CIG-3's**.

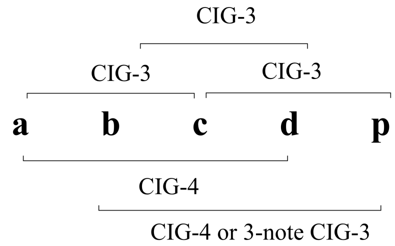
Extension rule for CIG-3's:

If CIG-3 (a-b-c) is extended by adding a note p that is the last note of CIG-3 (b-c-p), the resulting 4-note group (a-b-c-p) is always a CIG-4 or a 4-note CIG-3, except in some cases where $ic_{(b\text{-}c)} = 1$ and (a-b-c) is no instance of [3-1].

A formal (mathematical) proof of the extension rule for CIG-3's is given in Appendix 3. In this proof a distinction is made between CIG-3's not ending on a chromatic interval (Case 1: $ic_{(b\text{-}c)} \neq 1$) and CIG-3's that do Case 2: $ic_{(b\text{-}c)} = 1$). The second case is further divided in CIG-3's that are permutation of [3-1] (Case 2.2) and CIG-3's that aren't (Case 2.1).

5.4.3 From CIG-4 to CIG-5

When an extra note p is added to a CIG-4 (a-b-c-d) consisting of two CIG-3's (a-b-c) and (b-c-d), it is turned into a five-note group.[340] This five-note group has a structure of increased complexity; its consecutive notes consist at the same time of two CIG-4's (or a CIG-4 and a 4-note CIG-3 if note p = note b) and of three CIG-3's. Example 5.14 is a graphical representation of such a five-note group (a-b-c-d-p).[341]



Example 5.14: Graphical representation of the extension of CIG-4 (a-b-c-d) into a five-note group (a-b-c-d-p).

As in the extension of CIG-3's, to find out which extensions of CIG-4's lead to CIG-5's or 5-note CIG-4's, a first distinction is made between CIG-4's (a-b-c-d) ending on a non-chromatic interval (Case 1, $ic_{(c\text{-}d)} \neq 1$), and those ending on a chromatic interval (Case 2: $ic_{(c\text{-}d)} = 1$).

Let us first consider the case where the interval between the last two notes of the CIG-4 is non-chromatic ($ic_{(c\text{-}d)} \neq 1$). The ICV-rule tells us that—since (c-d-p) is a CIG-3—the value for ic 1 in the ic-vector of (c-d-p) is 1 or 2. Therefore either the interval between c and p or between d and p has to be chromatic ($ic_{(c\text{-}p)} = 1$ or $ic_{(d\text{-}p)} = 1$) or both. (a-b-c-d) is a CIG-4 and therefore the value for ic 1 in the ic-vector of (a-b-c-d) is 2 or 3 (again as a result of the ICV-rule). Since (a-b-c-d-p) is a superset[342] of (a-b-c-d) it contains the same interval classes between its notes a, b, c and d as (a-b-c-d). Since in addition either $ic_{(c\text{-}p)} = 1$ or $ic_{(d\text{-}p)} = 1$) (or both), The value for ic 1 in the ic-vector of (a-b-c-d-p) is at least 1 higher than that of (a-b-c-d). Therefore the value for ic 1 in the ic-vector of (a-b-c-d-p) is 3 or 4. From the ICV-rule follows that (a-b-c-d-p) is therefore always a CIG-5.

A formal proof for this assertion is given in Appendix 3. It is similar to the one given for the extension of CIG-3's. A more intuitive 'visualised proof' is given in Examples 5.15 a and b, showing the possible connections between notes in five-note groups (a-b-c-d-p) that are extensions with a CIG-3 (c-d-p) and a CIG-4 or 4-note CIG-3 (b-c-d-p) of CIG-4 (a-b-c-d), which in turn consists of two CIG-3's ((a-b-c) and (b-c-d)). Note that the proof makes no distinction between CIG-3's (b-c-d) that are no instances of [3-1] (shown in Example 5.15 a) and CIG-3's (b-c-d) that are instances of [3-1] (shown in Example 5.15 b). The horizontal lines in the grid of Example 5.15 a and b (and in the next similar examples) represent semitone intervals. The gap between the lower and upper part in Example 5.15 a indicates that the interval can be increased or reduced at this point.[343] As can be seen in Examples 5.15 a and b, all possible 'trajectories' from note a to note p result in either a CIG-5 or a 5-note CIG-4. The examples cover only ascending non-chromatic intervals c-d. For descending non-chromatic intervals, the trajectories have to be inverted.

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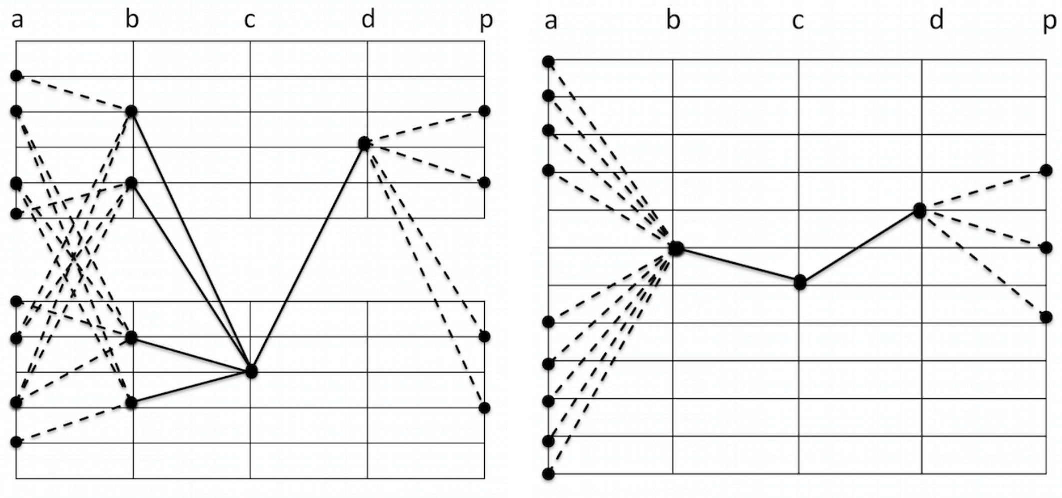
[340] Only extensions of CIG-4's are considered here. Extensions of 4-note CIG-3's are identical to extensions of other CIG-3's. These have been discussed in the previous section.

[341] a, b, c, d, and e are not pitch names (which are always written in capitals), but letter names indicating the position of the pitch classes within the five-note group. They may represent any pitch classes.

[342] A set A is a superset of set B, if B is a subset of A, that is: if B is contained in A.

[343] The separation between the upper and lower part of the grid can be extended to a maximum distance of ic 6 between notes c & d, and compressed to a minimum distance of ic 2 between c & d in Example 5.15, ic 3 between b & c in Example 5.16 a, and ic 2 between b & c in Example 5.16 b (as long as b-c-d is not turned into a 3-1). Trajectories in which notes c and d occur twice between notes a, b, c and d have been eliminated because only extensions of CIG-4's a-b-c-d are considered (necessarily four different pitch classes) and the croups c-d-e are CIG-3's (note that note e may be the same as note a or b, resulting in a 4-note CIG-5.

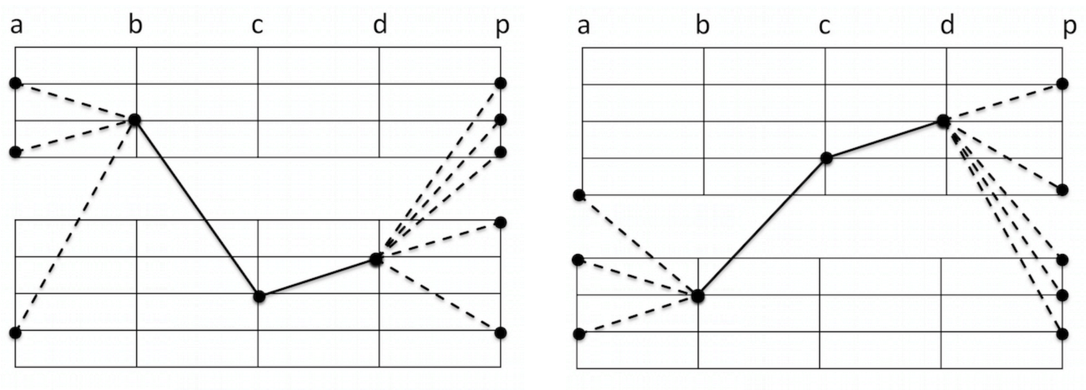


a b

Example 5.15: Possible extension of CIG-4 (a-b-c-d)
when interval between notes c and d is no ic1,
a) when CIG-3 (b-c-d) is no instance of [3-1],
and b) when (b-c-d) is an instance of [3-1].

The situation becomes more complicated when the initial CIG-4 (a-b-c-d) ends on a chromatic interval. In this case a distinction has to be made between initial CIG-4's that end on a CIG-3 (b-c-d) that is an instance of set class [3-1], and those that don't.

When (b-c-d) is no instance of [3-1], the formal proof in Appendix 3 demonstrates that the extension of CIG-4 (a-b-c-d) always results in either a 5-note CIG-4 (if note p is the same as either note a or b) or a CIG-5. This is due to the fact that note p is also part of CIG-4 (b-c-d-e), because it entails that note p is at ic 1 distance from at least one of the notes of that CIG-4. Likewise, note a is at ic 1 distance from at least one of the other notes of the CIG-3 (a-b-c) it belongs to. No new non-chromatic gap can be created by note p, and therefore the extension of CIG-4 always results in either a 5-note CIG-4 or a CIG-5. This intuitive proof is visualized in example 5.16 where, again, only ascending (chromatic) intervals between notes c and d are shown.

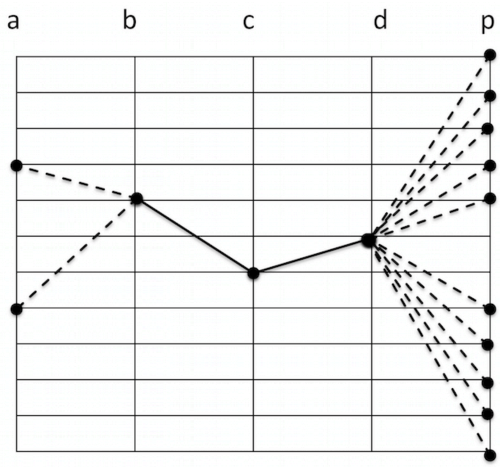


a b

Example 5.16: Possible extension of CIG-4 (a-b-c-d)
when $ic_{(c-d)} = 1$, but CIG-3 (b-c-d) is no instance of [3-1],
(a) with changing direction (down-up)
and (b) in one direction (up-up) for (b-c-d).

Let us now consider extensions of CIG-4 (a-b-c-d) in which CIG-3 (b-c-d) is a permutation of [3-1] and the interval between the last notes of the CIG-4 is chromatic ($ic_{(c-d)}$ = 1). A distinction has to be made here between CIG-3's (b-c-d) that consist of two consecutive interval classes in changing direction (a descending ic 2 followed by an ascending ic 1: [-2 +1], or vice versa: [+2 -1]), and CIG-3's (b-c-d) that consist of two semitone intervals in the same direction (either [+1 +1] or [-1 -1]).

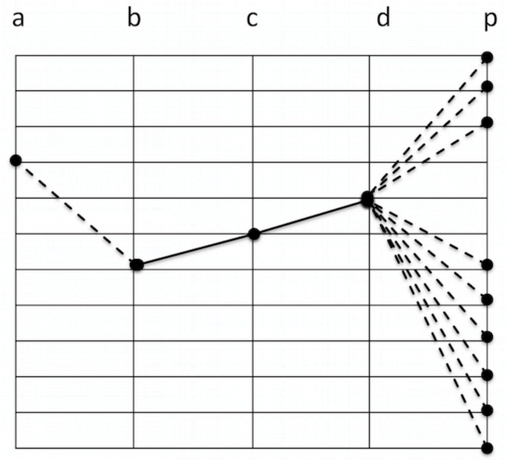
When CIG-3 (b-c-d) consists of two interval classes in changing direction, the interval between b and c is non-chromatic (it is indeed either +2 or -2, otherwise (b-c-d) would not be an instance of [3-1]). Since (a-b-c) is a CIG, note a has to be at ic 1 distance from b or c.[344] This means that there is no non-chromatic gap in CIG-4 (a-b-c-d). It is therefore a permutation of an instance of [4-1] and the value of ic 1 in its interval class vector is 3. Any extension of CIG-4 with note p would have a value of ic 1 in its interval vector that is at least 3 and would therefore be either a 5-note CIG-4 or a CIG-5 (in accordance with the ICV-rule). A visualisation of this intuitive proof is shown in Example 5.17. A formal proof is given in Case 2.2.1 of the proof for extensions of CIG-4's in Appendix 3.



Example 5.17: Possible extension of CIG-4 (a-b-c-d) ending on ic 1
when the CIG-3 (b-c-d) is a permutation of [3-1]
with changing directions (here: [-2 +1]).

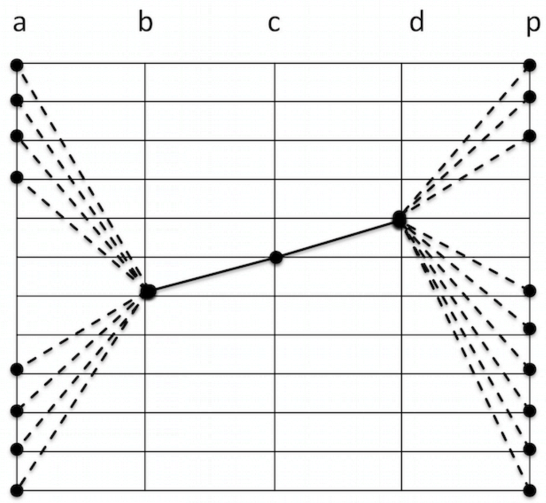
When CIG-4 (a-b-c-d) ends on a CIG-3 (b-c-d) that consists of two semitone intervals in the same direction (either [+1 +1] or [-1 -1]), the situation is similar to the previous one if (a-b-c-d) is a permutation of an instance of [4-1]. In that case, extensions of (a-b-c-d) are always either a 5-note CIG-4 or a CIG-5, as is shown in Example 5.18 and formally proved in Case 2.2.2.2 of the proof for the extension rule for CIG-4's in Appendix 3. Note that there is only one possibility for note a to form a CIG-4 (a-b-c-d) that is a permutation of an instance of [4-1]. The other possibility (a being a semitone lower than b) is excluded because then CIG-3's (a-b-c) and (b-c-d) would be the same CIG-3 (different only in transposition), which is impossible in CIG-series.

[344] In this case a cannot be at ic 1 distance from *both* b and c, otherwise note a would be the same as not d and (a-b-c-d) would not be a CIG-4.



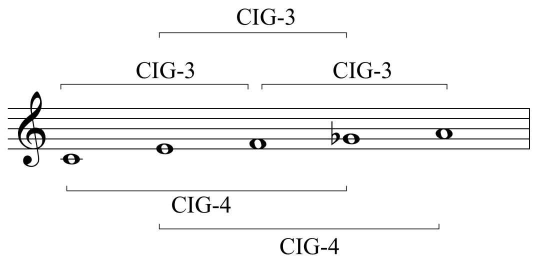
Example 5.18: Possible extension of CIG-4 (a-b-c-d) that is an instance of [4-1], where CIG-3 (b-c-d) is an instance of [3-1] in one direction (here: +1 +1).

Extensions of CIG-4 (a-b-c-d) can only result in a 5-note group that is not a 5-note CIG-4 or a CIG-5 in some cases where (a-b-c-d) (ending on a CIG-3 (b-c-d) consisting of two semitone intervals in the same direction) is not a permutation of an instance of [4-1]. Since in this case neither note a nor note p are necessarily at ic 1 distance from one of the notes of (b-c-d), there is a possibility that both form a non-chromatic gap with the other notes of the 5-note group. This is shown in Example 5.19 and formally proved in Case 2.2.2.1 of the proof of the extension rule for CIG-4's in Appendix 3.



Example 5.19: Possible extension of CIG-4 (a-b-c-d) that is no instance of [4-1], where CIG-3 (b-c-d) is an instance of [3-1] in one direction (here: +1 +1).

It is clear that in Example 5.19 some 'trajectories' from a to p do not result in either 5-note CIG-4's or CIG-5's. An example of a five-note group consisting entirely of CIG-3's and CIG-4's that is neither a CIG-5 nor a 5-note CIG-4 is shown in Example 5.20. In the five-note group of this example, there are two non-chromatic gaps (between C and E, and between G flat and A).



Example 5.20: A five-note group that is not a CIG-5,
although it consists exclusively of CIG-3's and CIG-4's.

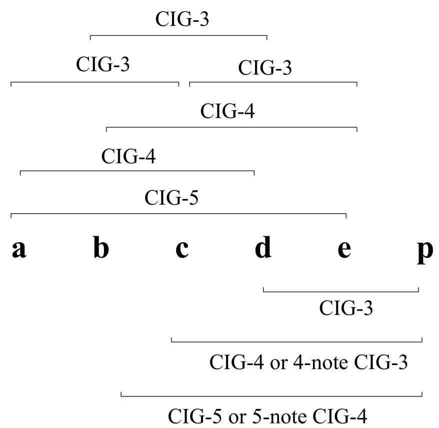
On the basis of the above considerations, a general rule for the extension of GIG-4's into five-note groups can be formulated as follows:

Extension rule for CIG-4's:

If CIG-4 (a-b-c-d) consisting of two CIG-3's (a-b-c) and (b-c-d) is extended by adding a note p that is the last note of CIG-3 (c-d-p) and of CIG-4 (b-c-d-p), the resulting 5-note group (a-b-c-d-p) is always a CIG-5 or a 5-note CIG-4, except in some cases where (b-c-d) is an instance of 3-1 with $ic_{(b-c)} = 1$ and $ic_{(c-d)} = 1$ (and both +1 or both -1)[345].

5.4.4 From CIG-5 to CIG-6 and higher cardinalities

When a CIG-5 (a-b-c-d-e)—obtained with the accumulative method described above (shown in Example 5.21)—is extended with an extra pitch class p that forms CIG's of cardinalities 3, 4 and (possibly) 5 with four of the preceding notes of the CIG-5 (CIG-3 (d-e-p), CIG-4 or 4-note CIG-3 (c-d-e-p) and CIG-5 or 4-note CIG-5 (b-c-d-e-p)), the resulting six-note group (a-b-c-d-e-p) (see Example 5.21) is always a CIG-6 or a 6-note CIG-5 (the latter occurs when note p is the same as note a, b or c).



Example 5.21: Graphical representation of the extension
of CIG-5 (a-b-c-d-e) into a six-note group (a-b-c-d-e-p).

Likewise, every extension of a CIG-n of order n higher than 5 with a note p that is the last note of a CIG-3, a CIG-4 and a CIG-5 always results in an (n+1)-note CIG-n or a CIG-(n+1). This results in the following general extension rule for CIG's of cardinality 5 and higher:

[345] Otherwise note b = note d and (b-c-d) is no CIG-3.

<u>Rule for extension of CIG-n to (n+1)-note group (for 5 ≤ n)</u>:

> If CIG-n (for 5 ≤ n) (z(n)-z(n-1)…z(2)-z(1)) consisting exclusively of unique CIG's of all cardinalities between 3 and n is extended by adding a note p that is the last note of unique CIG-3 (z(2)-z(1)-p), of CIG-4 (z(3)-z(2)-z(1)-p), and of CIG-5 (z(4)-z(3)-z(2)-z(1)-p), the resulting (n+1)-note group (z(n)-z(n-1)…z(2)-z(1)-p) is always a CIG-(n+1) or a (n+1)-note CIG-n.

A formal proof of this extension rule is given in Appendix 3. In this proof, again, a first distinction is made between CIG-n's that end on a chromatic interval (Case 1) and those that end on a non-chromatic interval (Case 2). In the former a further distinction is made between CIG-n's that end on CIG-3's that are instances of [3-1] (Case 2.2) and those that don't (Case 2.1). Ever further subdivisions culminate in a proof by mathematical induction.

Note that, although the order of CIG's in the rule for extensions of CIG's of order 5 and higher is not limited, in practice it is only relevant for CIG's up to order 8. Pitch class sets of order 10 to 12 are always CIG's; they all contain one non-chromatic gap at most and the value of ic 1 in their interval class vector is always one or two less than their order (e.g. the value of ic 1 in the interval class vector of pitch class sets of cardinality 10 is always 8 or 9).

5.4.4 General CIG-serialism

The ascertainment that CIG's have the lowest degree of tonality and prime consonance within all cardinality groups in combination with the above formulated extension rules for CIG's can now function as a starting point for the adaptation of CIG-serialism in order to obtain series with the highest possible degrees of atonality and dissonance.

A first question to answer is: if CIG-series have to consist exclusively of CIG's of all cardinalities, which CIG's should be used as 'building blocks' or basic 'structural units' for the series? The original technique was based on the combination of CIG-3's, but if also note-groups of higher cardinality have to be CIG's, is it appropriate or even possible to use CIG's of higher order as structural units or building blocks?

The answer is provided by the following consideration: there are twelve set classes representing CIG-4's (See Example 5.5). Each of these has 24 possible permutations. There are therefore 288 possible CIG-4's. Series using all CIG-4's once as structural unit would consist of 288 notes, which is extremely long. Moreover, not all of those CIG-4's consist of two CIG-3's (see Example 5.9) and the same CIG-3's occur in different CIG-4's, which would destroy the amotivity of CIG-series. This problem only intensifies with increasing cardinality of CIG's. Therefore the only possible structural units for CIG-series are CIG-3's, as was the case in the original CIG-3 technique.[346]

Even when CIG-3's are used as a structural unit, it is still possible to construct series that consist entirely of CIG's of higher order.[347] I call CIG-series in which all note-groups of consecutive notes of all cardinalities are CIG's (at least up to order 5) **general CIG-series**, and the serial technique based on such series **general CIG-serialism**. When henceforth the term 'CIG-serialism' is used, it refers to general CIG-serialism. The original CIG-technique is only a special case of CIG-serialism: CIG-3

[346] The possibility to use CIG-4's as structural units was explored during the construction of the series of *Danse du feu*. (see Section 9.4.3).

[347] The CIG's are not necessarily of *all* higher orders. The series of *A l'image du monde… originel* and *A l'image du monde… double,* for instance use only eleven pitch classes. CIG-12's do not occur in the series or scores of these two pieces (see Part 3).

serialism. So is CIG-3/4-serialism, the technique based on series consisting entirely of CIG-3's and 4's, but not necessarily of CIG-groups of higher cardinalities (as in Example 5.20).[348]

From the assessment of CIG-serialism (Section 5.2) we can conclude that general CIG-series have the lowest degrees of tonality and prime consonance. Their construction is more limited than that of CIG-3 or CIG-3/4 serialism. The limitations are determined by the following construction rules for (general)-CIG-series.

The original construction rules are preserved:

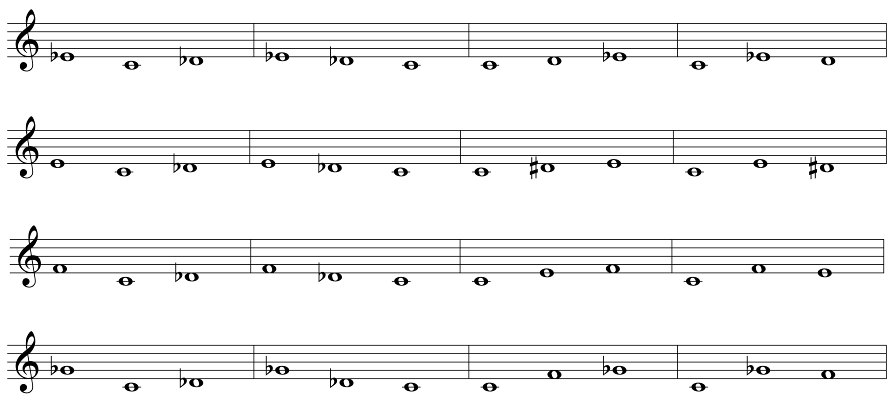
- CIG-series have CIG-3's as their unique elementary structural units.
- Each CIG-3 occurs exactly once in a CIG-series
- The series is 'closed' in order to preserve amotivity.

These three original construction rules are supplemented with two 'restriction rules'.

From the extension rule for CIG-3's we know that when two CIG-3's are combined (interwoven), the resulting four-note group is always a CIG-4 except possibly when the first CIG in the four-note group ends on ic 1 (this is never the case when the first CIG-3 is a permutation of an instance of [3-1]). This means that the extension of sixteen CIG-3's is limited according to the following restriction rule:

> **<u>First Extension Rule for general CIG-serialism</u>:**
>
> Whenever a CIG-3 (that is not a permutation of an instance of [3-1]) ending on ic 1 occurs in the construction of a series, the choice for the next CIG-3 is limited to those CIG-3's that result in a CIG-4. This means the additional note (the last note in the added CIG-3) has to be at ic 1 distance from at least one note of the preceding CIG-3.

The ordered prime form of all CIG-3's with restricted extension possibilities for the first restriction rule is shown in Example 5.22.

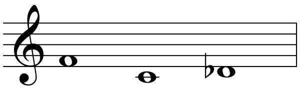


Example 5.22: CIG-3's with restricted extension possibilities for the first restriction rule.

When, for instance, in the construction of a CIG-series, the last added CIG-3 was F-C-D flat (see Example 5.23), the following CIG-3 (starting on C-D flat) has to end on either F sharp, F (resulting in

[348] *A l'image du monde… originel* and *A l'image du monde… double* are examples of CIG-3/4-serialism, but also coincidentally of general CIG-serialism (see Part 3).

a 4-note CIG-3), E, D or B. Any other added note would not result in a CIG-4 for the last four notes in the series.



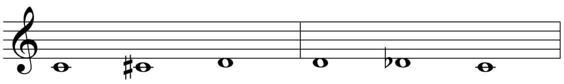
Example 5.23: A CIG-3 with limited extension possibilities.

The second restriction rule for the construction of CIG-series is determined by the fact that extensions of CIG-4's to CIG-5's with the addition of a last note of a unique CIG-3 do not always result in CIG-4's. The extension rule for CIG-4's shows that this may happen when the last CIG-3 is a permutation of an instance of [3-1] in one direction, either a succession of the ordered interval classes +1 +1 or -1 -1. This means that the extension of two CIG-3's is limited according to the following restriction rule:

<u>Second Extension Rule for general CIG-serialism</u>:

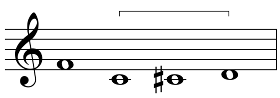
> Whenever a CIG-3 that consists of two consecutive ic 1-steps in the same direction occurs in the construction of a series, the choice for the next CIG-3 is limited to those CIG-3's that result in a CIG-5. This means that the additional note (the last note in the added CIG-3) has to be at ic 1 distance from at least one note of the preceding CIG-4.

The ordered prime form of all CIG-3's with restricted extension possibilities for the second restriction rule is shown in Example 5.24.



Example 5.24: CIG-3's with restricted extension possibilities
for the second restriction rule.

When, for instance, in the construction of a CIG-series, the last added CIG-3 was C-C sharp-D (see Example 5.25), the following CIG-3 (starting on C sharp-D) has to end on either F sharp, F (resulting in a 5-note CIG-4), E, D sharp, C (resulting in a 4-note CIG-3 and a 5-note CIG-4) or B. Any other last note would not result in a CIG-5.



Example 5.25: A CIG-4 containing a CIG-3 with limited extension possibilities.
The CIG-3 that causes the restricted extension possibilities is indicated.

As was proofed by the extension rule for CIG's or order 5 and higher, extensions of those CIG's always result in CIG's. Therefore, no additional restriction rules are required. Of the 54 CIG-3's, only 18 have restricted extension possibilities. If the two restriction rules are obeyed when these CIG-3's occur as the last CIG-3 in the construction of the series, the resulting series is always a general CIG-series, guaranteeing the lowest possible degree of tonality and prime consonance.

5.4.5 Additional remarks

5.4.5.1 Consonant intervals

The term ‘CIG’ is restricted to note groups of cardinality 3 or higher in the construction of the series. Ordered intervals (note groups of cardinality 2) are not included. Highly consonant intervals (e.g. ic 5 which has the highest possible degree of prime consonance $PC_{(1\text{-}5)} = 100$) occur in the series however, and this may disturb the constant degree of prime consonance of a piece. However, highly consonant intervals in the series are always surrounded by intervals that combine to form a CIG. Thus, highly consonant harmonic intervals can easily be avoided in the scores through implementation of the ‘chord rule’ discussed in Section 1.1.6. Whenever they would occur in a piece, the highly consonant harmonic interval can be made highly dissonant by adding one or several series notes preceding or succeeding the pitch classes of the consonant interval.

Notes 47 (A) and 48 (D) in the series of *Danse de l’eau et de l’air* for orchestra (2014), for instance, form ic 5 (see Example 5.26). The occurrence of this highly consonant harmonic interval has to be avoided in the score to keep the degree of consonance low.

Example 5.26: RHS for bar 213-216 of *Danse de l’eau et de l’air*,
showing rhythmic cells for series notes 47 and 48 (and the beginning of 49).

A first way to escape the occurrence of ic 5 as a harmonic interval is to avoid sounding the pc’s at the same time, which is possible in the case of *Danse de l’eau et de l’air* since in the RHS of this piece, they are never attacked at the same time. But even if the notes forming ic 5 (or another highly consonant interval) have simultaneous attacks in the RHS of a piece or if the piece is polyphonic at the moment where a highly consonant interval occurs in its RHS, the degree of consonance can always be kept low. In bars 214 and 215 of *Danse de l’eau et de l’air*, for instance, where notes 47 and 48 occur, the D is immediately accompanied by C sharp; at the end of bar 215, A is accompanied by B flat (see Example 5.27). As a result of the way CIG-series are constructed it is always possible to reduce the degree of prime consonance in this way.

Whereas passages with increased degrees of tonality and consonance, such as the major triad in bars 53 to 55 of my piece *Close my willing eyes* for three sopranos and ensemble (from 1999) shown in the Example 5.28, could occur, such passages become impossible or at least easily avoidable with the more rigorous technique of general CIG-serialism. Note combinations with increased degrees of tonality and prime consonance are absent in the series, and careful combinations of series (such as the simultaneous use of semi-tone transpositions of the series, as is the case in *Le sourire infini des ondes*)[349] help avoiding passages with higher T- and PC-values.

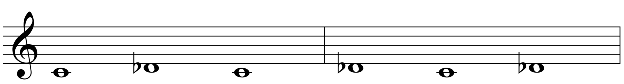
[349] See Example 9.15 in Part 3.

Example 5.27: RHS first and 2nd violin parts of bar 213-216 from *Danse de l'eau et de l'air*.

Example 5.28: Bars 51-55 of *Close my willing eyes* for three sopranos and ensemble (1999).
Note the occurrence of the triad D major in bars 53 to 55.

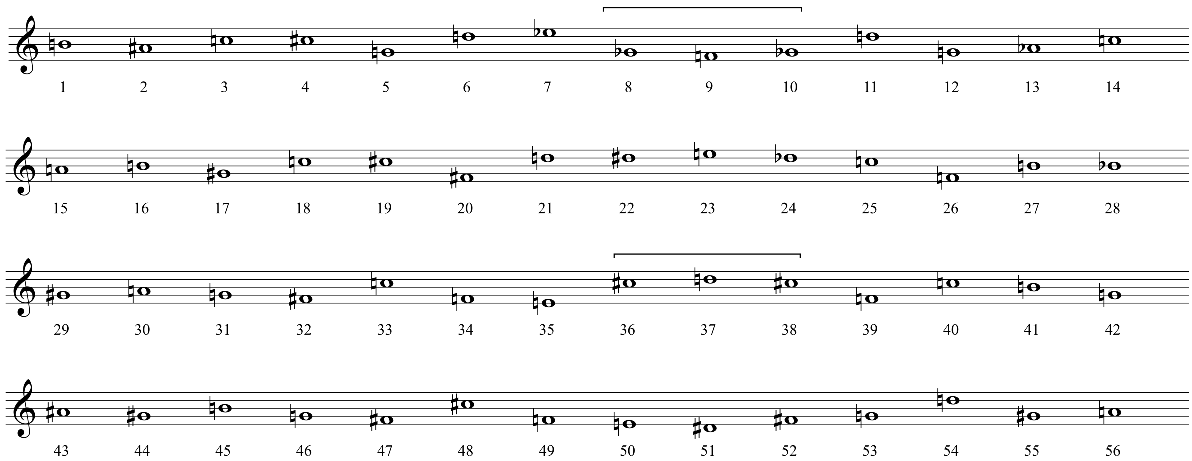
5.4.5.2 56-CIG-serialism

There is one special case of chromatic three-note group of order 2 (with two different pitch classes): the note group consisting of an ascending ic 1 (+1) followed by a descending ic 1 (-1) or vice versa (-1 +1). Although the concept of CIG is only defined for note groups of cardinality 3 or higher, these three-note groups may be considered as special cases of CIG's: 3-note CIG-2's (see Example 5.29).



Example 5.29: two instances of 3-note CIG-2's.

54-CIG serialism can be extended to 56-CIG-serialism by including the two 3-note CIG-2's in the series. Since the 3-note CIG-2's do not cause an extension of the number of different pitch classes in the series (i.e. to the cardinality of the series at a certain point, the degree of tonality or prime consonance of the series (or the music composed with it) is not affected with the occurrence of the 3-note CIG-2's. Therefore the construction rules for 54-CIG-series—including the two extension rules—remain unchanged in the case of 56-CIG-serialism. The orchestral pieces *Danse du feu* (2012), and *Danse de l'eau et de l'air* (2014) are examples of pieces composed with a 56-CIG-series. Example 5.30 shows the series of *Danse du feu*. The two 3-note CIG-2's are indicated with horizontal square brackets.



Example 5.30: The 56-CIG series of *Danse du feu*.
The two 3-note CIG-2's are indicated with square brackets.